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(Pages : 3)

Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY)
EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 05—NUMBER THEORY

(2019 Admissions)

Time : Three Hours

Maximum : 30 Weightage

General Instructions.

Part A

Answer all questions.
Each question has weightage 1.

1. Find all integers n such that :

$$\phi(n) = n/2.$$

2. If $n \geq 1$, prove that :

$$\Lambda(n) = \sum_{d|n} \mu(d) \cdot \log \frac{n}{d}.$$

3. Prove that $[2x] - 2[x]$ is either 0 or 1.

4. For $x > 0$, prove that :

$$0 \leq \frac{\psi(x)}{x} - \frac{\mathcal{J}(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x} \log 2}.$$

5. For all $x \geq 1$, prove that :

$$\sum_{p \leq x} \frac{\log p}{p} = \log x + O(1).$$

6. Evaluate the Legendre' symbol $(73 | 383)$.

7. Prove that Legendre's symbol $(n | p)$ is a completely multiplicative function n of n .

8. Briefly describe about digraph transformation.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any ~~six~~ questions by choosing two questions from each unit.
Each question carries a weightage of 2.

Unit I

9. For $n \geq 1$, prove that :

$$\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right).$$

10. State and prove the selberg identity.

11. If $x \geq 1$, prove that :

$$\sum_{n > x} \frac{1}{n^s} = O(x^{1-s}) \text{ if } s > 1.$$

Unit II

12. Prove that the relation :

$$\lim_{x \rightarrow \infty} \frac{\pi(x) \log x}{x} = 1$$

implies the relation :

$$\lim_{x \rightarrow \infty} \frac{J(x)}{x} = 1.$$

13. Let $\{a(n)\}$ be a non-negative sequence such that :

$$\sum_{n \leq x} a(n) \left[\frac{x}{n} \right] = x \log x + O(x) \text{ for all } x \geq 1.$$

For $x \geq 1$, prove that :

$$\sum_{n \leq x} \frac{a(n)}{n} = \log x + O(1).$$

14. State and prove the Abel's identity.

Unit III

15. For every odd prime p , prove that :

$$\left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8}$$

16. Describe briefly about RSA cryptosystem.

17. In the 27-letter alphabet (with blank = 26), use the affine enciphering transformation with key $a = 13, b = 9$ to encipher the message "HELP ME".

(6 × 2 = 12 weightage)

Part C

*Answer any two questions.
Each question carries a weightage of 5.*

18. Show that the set of all arithmetical functions f with $f(1) \neq 0$ forms an abelian group with respect to the Dirichlet product.

19. For every integer $n \geq 2$, prove that :

$$\frac{1}{6} \frac{n}{\log n} < \pi(n) < 6 \frac{n}{\log n}$$

20. If p and q are distinct odd primes, then prove that :

$$\left(\frac{p}{q}\right) \left(\frac{q}{p}\right) = (-1)^{(p-1)(q-1)/4}$$

21. The message "KVW ? Ta!KJB ? FVR ." (The blanks after ? and R are part of the message, but the final . is not) is intercepted. It is known that a linear enciphering transformation is being used with a 30-letter alphabet, in which A – Z have numerical equivalents 0 – 25, blank = 26, ? = 27, ! = 28, . = 29. It is further known that the first six letters of the plaintext are "CIA". Find the deciphering matrix A^{-1} and the full plaintext message.

(2 × 5 = 10 weightage)