C 61267 (Pages 4) Name

Reg. No. . . . .

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2014

(CUCSS)

Mathematics

MT 4E 05—OPERATIONS RESEARCH

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Give an example to show that spanning tree of a graph need not be unique.

2. Write the problem of maximum flow in the generalized form.

3. Tasks A, B, C, . . H, I constitute a project. The notation X < Y means that the task X must be finished before Y can begin. With this notation,

   A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I,

   draw a graph to represent the sequence of tasks.

4. Describe the effect of introducing the constraint $3x_1 - 2x_2 \quad 2$ in the L.P. problem

   Minimize $Z = 4x_1 + 5x_2$

   subject to
   $2x_1 + x_2 \quad 6$
   $x_1 + 2x \ 5\_ \ 5$
   $x_1 + x_2 \geq 1$
   $x_1 + 4x_2 \quad 2$
   $x_1, x_2 \quad 0.$

   Whose optimal solutio 1 is $x_1 = 2/3$, $x_2 = 1/3$.

5. What do you mean by parametric programming ?

6. Let $f(X)$ be a real-valued function in E_n, G (X) a vector function consisting of real-valued functions $g_i(X)$, $i = 1, 2, \ldots, m$ as components and

   $$F(X, Y) = f(X) + Y' G(X)$$

   where Y is a vector in E_m. If F (X, Y) has a saddle point (X_0, Y_0) for every Y 0, prove that X_0 is a minimum of $f(X)$ subject to the constraints G (X) 5_ 0.

Turn over

7. Write the Kuhn-Tucker conditions for the problem :

Minimize $f = x_1^2 + x_2^2$

subject to $x_1 + x_2$ 4

$2x_1 + x_2$ 5

8. What is the advantage of solving the dual problem in a geometric programming problem.
9. What is the difference between a posynomial and a polynomial.
10. Describe a method of dynamic programming to solve the problem

Maximize $j = 1$ $f_j (u_j$

subject to $\sum_{j=1}^{n} a_J u_J = k$ $u_j > 0$, $a_j > 0$

11. Define the term forward recursion as used in dynamic programming.
12. Solve by the method of dynamic programming

Maximize $\phi_2 = f_2\ f_1$ where $f_1 = u1$, $f_2 = u_2$

subject to 1 <u1 s3, -1<u2 1.

13. Show that the function $f(x) = x^2$, O x < 1 is unimodal in (0, 2).
14. Find the minimal point of $x^3 - 3x + 2$, 0 5 x 3 by Newton-Raphson method.

(14 x 1 = 14 weightage)

Part B

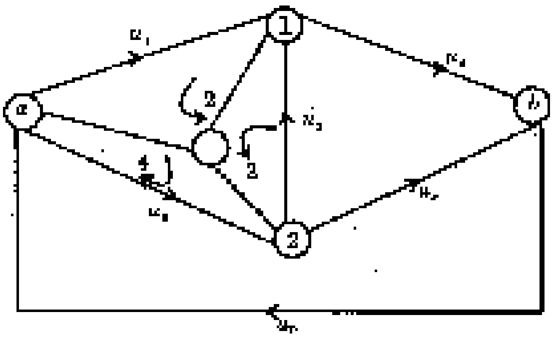
Answer any **seven** *questions.*
Each question has weightage 2.

15. Define the following terms :

(a) Tree ; (b) Spanning Tree.

16. Find the maximum flow in the following graph with the constraints

$2 \le x_1\ 10,\ 4 \le x_2\ 12,\ -2 \le x_3 \le 4,\ 0 \le x_4 \le 5,\ 0 \le x_5 \le 10$



17. **Describe the effect of introducing new variables on the optimal solution of an L.P. problem.**

18. **Solve graphically :**

Maximize $(x_1 - 4)^2 + (x_2 - 4)^2$

subject to $x_1 + x_2 \le 6$

$x_1 - x_2 \quad 1$

$2x_1 + x_2 \quad 6$

$\frac{1}{2} x_1 - x_2 \quad -4$

$0,\ x_2 \quad 0.$

19. **State Kuhn-Tucker theorem.**

20. **Write the orthogonality conditions in a general geometric programming problem.**

21. **What are the essential features of dynamic programming problem.**

22. **Minimize : $u_1^2 + u_2^2 + u_3^3$**

subject to $u_1 + u_2 + \quad 10$

$u_1,\ U2\ U3 \quad a'\ O.$

23. Briefly describe the Fibonacci search plan.

24. Find the minimal point of $x^3 - 3x + 2,\ O < x < 3$ by quadratic interpolation.

(7 x 2 = 14 weightage)

Turn over

Part C

Answer any two questions.
Each question has weightage 4.

25. A project consists of activities A, B, C, . . , M. In the following data, X – Y = C means Y can start after C days of work on X. A, B, C can start simultaneously. K and M are the last activities and take 14 and 13 days repectively.

A – D = 4, B – F = 6, B – E = 3, C – E = 4, D – H = 5, D – F = 3, E – F = 10, F – G = 4, G – I = 12, H-I= 3,H-J= 3,J-K= 8,I-K= 7,L-M= 9.

Find the least time of completion of the project.

26. A factory can manufacture two products A and B. The profit on a unit of A is Rs. 80 and of B is Rs. 40. The maximum demand of A is 6 units per week, and of B it is 8 units. The manufacturer has set up a goal of achieving a profit of Rs. 640 per week. Formulate the problem as goal programming, and sovle it.

27. Solve by the method of quadratic programming :

Minimize $-6x_1 + 2x_1^2 - 2x_1 x_2 + 2x_2^2$
subject to $x_1 + x_2 \le 2,$
0, X2 O.

28. Find the maximum of f *(x)* = – 0.55 + 3x – x^2 by Rosenbrock algorithm starting from $x = 0$, $h = 1$.

(2 x 4 = 8 weightage)