

**FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016**

(CUCSS)

Mathematics

MT 4E 05—OPERATIONS RESEARCH

Time : Three Hours

Maximum : 36 Weightage

**Part A***Answer all questions.**Each question has weightage 1.*

1. Briefly describe the minimum path problem.
2. For any feasible flow  $\{x_i\}$ ,  $i = 1, 2, \dots, m$  in the graph, prove that the flow  $x_{ij}$  in the return arc is not greater than the capacity of any cut in the graph.
3. State the problem of maximum potential difference in a network.
4. Describe the effect of deletion of Variables on the optimal solution of an LP problem.
5. What is goal programming.
6. Let  $f(x)$  be a real-valued function in  $E_n$ ,  $G(X)$  a vector function consisting of real valued functions  $g_i(x)$ ,  $i = 1, 2, \dots, m$  as components and  $F(X, Y) = f(x) + Y' G(X)$  where  $Y$  is a vector in  $E_m$ . If  $F(X, Y)$  has a saddle point  $(X_0, Y_0)$  for every  $Y \geq 0$ , prove that  $X_0$  is a minimum of  $f(x)$  subject to the constraints  $G(X) \leq 0$ .
7. Write Kuhn-Tucker conditions for the problem :

$$\text{Minimize } 16(x_1 - 2)^2 + (4x_2 - 9)^2$$

$$\text{subject to } x_1 - x_2 \geq 0$$

$$x_1 + x_2 \leq 6,$$

$$x_1, x_2 \geq 0.$$

8. What is the advantage of solving the dual problem in geometric programming problems.
9. Write the general form of a geometric programming problem.
10. Define the decision variables and the state variables in a dynamic programming problem.
11. Explain the difference between forward recursion and backward recursion in dynamic programming.

**Turn over**

12. Show that the function  $f(x) = x^2$ ,  $0 \leq x \leq 1$  is unimodal in  $(0, 2)$ .
13. What is meant by a sequential search plan.
14. Find the minimal point of  $x^3 - 3x + 2$ ,  $0 \leq x \leq 3$ , taking  $\epsilon = 0.01$  by the method of false position.

(14 x 1 = 14 weightage)

**Part B**

*Answer any seven questions.  
Each question has weightage 2.*

15. Briefly describe an algorithm to find the spanning tree of minimum length if the length of each arc of the graph is given as a non-negative number.
16. A project consists of activities A, B, C, —M. In the following data  $X - Y = C$  means Y can start after C days of Work on X. A, B, C can start simultaneously. K and M are the last activities and take 14 and 13 days respectively. A - D = 4, B - F = 6, B - E = 3, C - E = 4, D - H = 5, D - F = 3, E - F = 10, F - G = 4, G - I = 12, H - I = 3, H - J = 3, J - K = 8, I - K = 7, I - L = 7, L - M = 9. Find the least time of completion of the project.
17. For the problem :

$$\begin{aligned} &\text{Maximize } f = x_1 - x_2 + 2x_3 \\ &\text{subject to } \quad -x_2 + x_1 \leq 4, \\ &\quad \quad \quad x_1 + x_2 - x_3 \leq 3 \\ &\quad \quad \quad 2x_1 - 2x_2 + 3x_3 \leq 15 \\ &\quad \quad \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

assuming  $x_4, x_5, x_6$  respectively as the slack variables for the three constraints, the optimal table is the following :

Basis	Values	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$
$x_3$	21	4		1		2	1
$x_4$	7	2			1	1	0
$x_2$	24	5	1			3	1
$-f$	18	2				1	1

Carry out sensitivity analysis when the coefficient of  $x_1$  in the objective function changes to 2.

8. Minimise  $f = x_1^2 + x_2^2$   
subject to  $(x_1 - 1)^2 - x_2^2 > 0$ .

9. Mention briefly an algorithm for solving a quadratic programming problem.
10. Explain the terms weight functions and normalised weight functions in geometric programming problems.

21. Describe a method of dynamic programming to solve the problem  $z = \sum_{j=1}^n f_j(u_j)$

subject to  $\sum_{j=1}^n a_j u_j = b$

$a_j, b \in \mathbb{R}, a_j \geq 0, b > 0$

22. Minimize  $u_1 + 2u_2 + u_3$

subject to  $u_1 + u_2 + u_3 = 10, u_1, u_2, u_3 \geq 0$

by forward recursion.

23. Briefly describe the computational algorithm for Fibonacci search plan.

24. Find the point of minimum of  $f(x) = e^{-x} + x^2$  in the interval (0, 1) using golden section method.

(7 x 2 = 14 weightage)

### Part C

Answer any two questions.  
Each question has weightage 4.

25. Find the maximum non-negative flow in the network described below, arc  $(V_j, V_k)$  being denoted as  $(j, k)$ ,  $V_s$  is the source and  $V_t$  is the sink.

Arc	(a, 1)	(a, 2)	(1, 2)	(1, 3)	(1, 4)	(2, 4)	(3, 2)	(3, 4)	(4, 3)	(3, t)	(4, t)
Capacity	8	10	3	4	2	8	3	4	2	10	9

26. Minimize  $f(X) = (1 + X) x_1 + (-2 - 2x_2) x_2 + (1 + 5x_3) x_3$ ,

subject to  $2x_1 - x_2 + 2x_3 \leq 2$ ,

$$x_1 - x_2 \leq 3$$

$$x_1 + 2x_2 - 2x_3 \leq 4$$

$$x_1, x_2, x_3 \geq 0$$

27. Minimize  $x_2^2 + x_3 - 5x_1$

subject to  $5x_1 - 3x_2 + x_3^2 \leq 2$

the variables being all positive.

28. Maximize  $\sum_{n=1}^4 (4u_n - nu_n)$

subject to  $\sum_{n=1}^4 u_n = 10, u_n \geq 0$ .

(2 x 4 = 8 weightage)