C 42501 (Pages : 3) Name.......................................

Reg. No....................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2013

(CUCSS)

Mathematics

MT 4C 15—FUNCTIONAL ANALYSIS—II

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Each question carries a weightage of 1.

1. Let X, Y and Z be normed spaces. Show that if $F : X \to Y$ is continuous and $G : Y \to Z$ is closed, then Go F $X \to Z$ is closed.

2. Let X_0 be a dense subspace of a normed space X. **Show that there is a linear isometry from X' onto X_0'**

3. Let X denote the sequence space l^∞. Let II $\|'$ be a complete norm on X such that if $\|X_n$ 0, then x_n *(s) – x (s)* for every $s = 1, 2, ...$ show that II II' is equivalent to the usual norm 0 $\|_2$ on X.

4. Let (K_n) be a sequence of eigen values of a bounded **linear operator on a normed space X. Show that if** $K_n \to k$ in K, then *k is* **an approximate eigen value of A.**

5. **Show that dual of a separable reflexive normed space is separable.**

6. **Give an example of a separable normed space which is not reflexive.**

7. **Give an example of a continuous map on a normed space which is not compact.**

8. **Let E be a subset of a Hilbert space H over K. Show that $E^\perp$ 1 is the closure of the span of E.**

9. **Let H be a Hilbert space over K and AεBC (H). Show that VII = $\|A$**

10. **Show that if X is an inner product space and if AεBL (X), then there may not exist BεBL (X) such that (A (x), y) = (x, B (y)) for all** *x, y* **c H.**

11. **Let H be a Hilbert space and AεBL (H). Show that A is normal if $\|A(x)\| = \|A^*(x)\|$ for all** x c **H.**

12. **Let H be a Hilbert space and AεBL (H) be normal. Show that if x_1 and x_2 are eigen vectors of A corresponding to distinct eigen values , then x_1 1 x_2.**

Turn over

13. Let H be a Hilbert space and $A \in BL(H)$ be self adjoint. Show that $A^2 > 0$ and $A \leq \|A\| I$.

14. Give an example of Hilbert-Schmidt operator on the Hilbert space $H = l^2$.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. Let X be normed space over K and f X K be linear. Show that f is closed iff f is continuous.

16. Let X and Y be a Banach spaces and $F \in BL(X, Y)$. Show that R (F) is linearly homeomorphic to (F) if and only if **R (F)** is closed in Y.

17. Let X and Y be normed spaces and $F \in BL(X, Y)$. Show that $\|F'\|_{II} = \|F\|$ and $F'' J_X = J_Y F$, **where** J and J_Y are the canonical embeddings of X and Y into **X" and Y" respectively.**

18. Show that if $1 < p < \infty$, **then** l **is reflexive.**

19. Let X be a Banach space. Show that the set of all compact operators on X is closed in BL (X).

20. Let H be a Hilbert space and F be a non-empty closed subspace **of H.** Show **that** $H = F + F^{\perp}$.

21. Let H be a Hilbert space and $A \in BL(H)$. **Show that if A is self-adjoint, then** $\|A\| \sup \{|\langle A(x), x\rangle| : x \in H, \|x\| \leq 1\}$.

22. Let H be a Hilbert space and $A \in BL(H)$. **Show that if A is unitary, then for every orthonormal basis** $\{u_\alpha\}$ **of H,** $\{A(u_\alpha)\}$ **and** $\{A^*(u_\alpha)\}$ **are both orthonormal bases for H.**

23. Let H be a non-zero Hilbert space and $A \in BL(H)$ **he self-adjoint. Show that** $\{m_A \; M_A\}$ **cad (A) = a (A)** $\subseteq [m_A \; M_A]$.

24. Let H be a Hilbert space and $A \in BL(H)$. **Show that** if A is compact, then A* is also compact.

(7 x 2 = **14** weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of **4.**

25. Let X be normed space and $A \in BL(X)$ be of finite rank. Show that $\sigma_e(A) = a_s(A) = a(A)$.

26. Let X be a Banach space which is uniformly convex in some equivalent norm. Show that X is reflexive.

27. State Riesz representation theorem. Show that the Riesz representation theorem does not hold for an incomplete inner product space.

28. Let **H** be a finite dimensional Hilbert space over K and $A \in BL(\mathbf{H})$. Suppose that K = C and A is normal, or that K = R and A is self adjoint. Show that there is an orthonormal basis for **H** consisting of eigenvectors of A.

(2 x 4 = 8 weightage)