D 31326 (Pages : 3) Name..................................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2012

(CUCSS)

Mathematics

MT 3C 12—FUNCTIONAL ANALYSIS—I

(2010 admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *question&*
Each question carries 1 weightage.

1. Prove or disprove : A sequence (x_n) in the metric space $l^p, 1 \le p \le \infty$, converges to x in if $x_n(j) \to x(j)$ for each $j = 1, 2, 3, \ldots$
2. Give an example of a bounded sequence in a metric space which is not cauchy.
3. State Minkowski's inequality for measurable functions on a measurable subset of R.
4. Define nth Dirichlet Kernal D_n and show that $\int D_n(t)\, dt = 2\pi$
5. Let Y be a subspace of normed space X. Show that is a normed space.
6. Define inner product space.
7. State Gram-Schmidt orthonormalization theorem.
8. Let (x_n) be a sequence in a Hilbert space H. Show that if $\sum_{n=1}^{\infty} \| x_n \| < \infty$, then $\sum_n x_n$ converges in H.
9. Let X be an inner product space. Let $E \subset X$ and $x \in E$. Show that there exists a best approximation from E to x if $x \in E$.
10. Let X be a normed space, $f \in X'$ and $f \ne 0$. Let $a \in X$ with $f(a) = 1$, $r > 0$. Show that $U(a, r) \cap Z(f) \ne 0$ iff $\| f \| \ge \frac{1}{r}$.
11. Show that C_{00} is not closed in l^∞
12. What is the geometrical interpretation of the uniform boundedness principle ?

Turn over

13. Let X be a normed space over K and $x \in X$. Define j_x K by $f_x(f) = f(x)$ for $f \in$. Show that $j_x \in X''$ and II ix 11'11 x11.

14. Let X be a normed space and (x_n) be a sequence in X such that $(f(x_n))$ converges in K for every $f \in X$ • Show that the sequence (x_n) is bounded.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 weightage.

15. Show that a non-empty subset of a separable metric space is separable in the induced metric.

16. State and prove Riemann–Lebesgue lemma.

17. Prove that the three norms $\| \|_1$, $\| \|_2$ and $\| \|_\infty$ are equivalent.

18. Let X and Y be normed spaces and Z be a closed subspace of X. Show that if $F \in BL(X/Z, Y)$ and we let $F(x) = (x + Z)$ for $x \in X$, then $F \in BL(X, Y)$ d F H

19. Let $<, >$ be an inner product on a linear space X and T x X be a linear one-to-one map. Let : $< x, y >_T = < T(x), T(y) >$ for $x, y \in X$.

Show that $<, >_T$ is an inner product on X.

20. State and prove Bessel's inequality.

21. Let $X = C([-1,1])$, $x(t) = 1 - t$, $x_1(t) =$ and $x(t) = \cos \pi t$ for t —1,1}. Show that the best approximation from span $\{x_0, x_1\}$ to x is $4x_1$

22. Let Y be a subspace of a normed space X and $a \in X$ but $a \notin Y$. Show that there is some $f \in$ such that $f/_Y = 0$, $f(a) = \text{dist}(a, \bar{Y})$ and $\| f \| = 1$.

23. Show that a normed space X in a Banach space iff every absolutely summmable series of elements in X is summable in X.

24. Let X be a normed space and E be a subset of X. Show that E is bounded in X iff f (E) is bounded in K for every $f \in X$.

= 14 weightage)

Part C

Answer any two questions.
Each question carries **4** *weightage.*

25. **Show that for 1 $p < \infty$, the metric space l^p is separable, but l^∞ is not separable.**
26. **Show that every finite dimensional subspace of a normed space X is closed in X.**
27. **Show that a non-zero Hilbert space H is separable if H has a countable orthonormal basis.**
28. **Let. X be a normed space. Show that for every subspace Y of X and every $g \in Y'$, there is a unique Hahn–Banach extension of g to X if X is strictly convex.**

2 X 4 = 8 weighatge)