D 6715 (Pages : 3) Name..................................

Reg. No..............................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 3C 11—COMPLEX ANALYSIS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Prove that an analytic function in a region f2 whose derivative vanishes identically must reduce to a constant.

2. Find the linear transformation which carries $0, i, -i$ into 1, —1, 0.

3. Find the fixed points of the linear transformation $w = \frac{2z}{3z-1}$

4. Describe the Riemann surface associated with the function $w = z^n$, where $n > 1$ is an integer.

5. Compute $\int_\gamma x\, dz$ where γ is the directed line segment from 0 to $1 + i$.

6. If the piecewise differentiable closed curve γ does not pass through the point a, then prove that the value of the integral $\int_\gamma \frac{dz}{z-a}$ is a multiple of $2\pi i$.

7. Compute $\int_{|z|=1} \frac{e^z}{z}\, dz$

8. Show that e^z have essential singularity at co.

9. Find the poles and residues of $\frac{1}{z^2 + 5z + 6}$

Turn over

10. If $p(z)$ is a non-constant polynomial, prove that there is a complex number a with $p(a) = 0$.

11. Define Harmonic function.

12. State Mean Value property.

13. Obtain the Taylor series which represents the function $\frac{z^2 - 1}{(z + 2)(z +}$ in the region $|z| < 2$.

14. Prove that there does not exist an elliptic function with a single simple pole.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Prove that the cross ratio (z_1, z2, z3, z4) is real if and only if the 4 points line on a circle *or* on a straight line.

16. Find the linear transformation which carries $|z| = 1$ and $\left| z \quad 1 \right| = \mathbf{1}$ into concentric circles. What is the ratio of the radii.

17. If $f(z)$ is analytic in an open disk A , then prove that :

$(z)\, dz = 0$ for every closed curve y in A .

18. Suppose that $f(z)$ is analytic in an open disk A , and let y be a closed curve in A . For any point a not on y , prove that :

$$n(r, a) f(a) = \frac{1}{2\pi i} \int_r \frac{f(z)}{z - a} dz.$$

where $n(y, a)$ is the index of a with respect to γ.

19. Show that any function which is meromorphic on the extended plane is rational.

20. Prove that a non-constant analytic function maps open sets onto open sets.

21. State and prove Argument principle.

22. If the functions $f_n(z)$ are analytic and non-zero in a region SI, and is $f_n(z)$ converges to *f (z),* uniformly on every compact of Ω, prove that *f (z)* is either identically zero or never equal to zero in S2.

23. If u is harmonic, show that $f = u_x - 1u_y$ is analytic.

24. Show that any even elliptic function with periods w_1 and w_2 can be written in the form :

$$C \prod_{k=1}^{n} \frac{\mathcal{P}(z)-\mathcal{P}(a_k)}{\mathcal{P}(z)-\mathcal{P}(b_k)}$$

provided that 0 is neither a zero nor a pole.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries 4 *weightage.*

25. List *f (z)* be analytic on the set R^1 obtained from a rectangle R by omitting a finite number of interior points If :

$\lim_{z} (z - \zeta_j) f(z) = 0$ for all *j*. then prove that J f *(z)* $dz = 0$.

OR

26. A region Ω is simply-connected if and only if $n(\gamma, a) = 0$ for all cycles yin S2 and all points a which do not belong to S2.

27. Describe the Laurent series development.

28. Derive the Poisson Integral Formula for Harmonic Functions.

(2 x 4 = 8 weightage)