

D 6717

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Name.....

Reg. No.....

**THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016**

(CUCSS)

Mathematics

MT 3C 13—TOPOLOGY—II

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** questions.

Each question has *weightage* 1.

1. Distinguish between box and large box in product space.
2. Define choice function. Do choice functions necessarily exist ?
3. Define standard sub-base and standard base for the product Topology.
4. Distinguish between locally connectedness and path connectedness.
5. Define the topology of **pointwise** convergence.
6. Prove that the intersection of any families of boxes is a box.
7. Prove that a subset of X is a box if it is the intersection of a family of walls.
8. Define first **Homotopy** group of a Topological space.
9. In a simply connected space X, prove that any *two* paths having the same initial and final points are path **homotopic**.
10. Prove that continuous image of a **countably** compact space is **countably** compact.
11. State the embedding lemma in topological spaces.
12. Distinguish between **Alexandroff compactification** and **Stone-Cech compactification** of a space X.
13. Define totally bounded subset of a space. Prove that subset of a totally bounded set is totally bounded.
14. Define metrically topologically complete space. Write an example for the same.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** questions.

Each question has *weightage* 2.

15. Justify the term 'box' geometrically for the products of copies of the real line.
16. Let  $C_i$  be a closed subset of the space  $X_i$  for  $i \in I$ . Then prove that  $f \in \prod_{i \in I} C_i$  is a closed subset of  $\prod_{i \in I} X_i$  with respect to the product topology.

Turn over

17. Prove that the projection functions are open.
18. Prove that a topological product is regular if and only if each coordinate space is regular.
19. Prove that the evaluation function of a family of functions is continuous if and only if each member in the family is continuous.
20. Prove that the fundamental group of unit ball in  $\mathbb{R}^n$  is trivial.
21. Prove that a metric space is compact if and only if it is **countably** compact.
22. Prove that every locally compact, **Hausdorff** space is regular.
23. Prove that every compact metric space is complete.
24. Prove that a non-empty complete metric space is of second category.

(7 x 2 = 14 weightage)

### Part C

*Answer any two questions.  
Each question has **weightage** 4.*

25. Prove that any continuous real-valued function on a closed subset of a normal space can be extended continuously to the whole space.
26. Prove that a subset of  $X$  is a large box if and only if it is the intersection of finitely many walls.
27. Prove that **metrisability** is a **countably** productive property.
28. Prove that a metric space is compact if and only if it is complete and totally bounded.

(2 x 4 = 8 weightage)