**D 71327**  (Pages : 3)  Name.........................................

Reg. No...................................

# THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

### Mathematics

### MT 3C 11—COMPLEX ANALYSIS

Time : Three Hours  Maximum : **36 Weightage**

## Part A

*Answer* **all** *questions.*
*Each question carries 1 weightage.*

1. Show that if a linear transformation has $\infty$ for its only fixed point, then it is a translation.

2. State the symmetry principle.

3. Compute the cross ratio :

   $(i, 0, -1, \infty)$.

4. Find the image of the hyperbola $\{z = x + iy : x^2 - y^2 = 1\}$ under the map $f(z) = z^2$.

5. State the Cauchy's Integral Formula.

6. Compute $\int_r dz$, where $r(t) = e^{it}, 0 < t < 2\pi$

7. What is the nature of the singularity of $e^z$ at $z = \infty$ ?

8. State general form of Cauchy's theorem.

9. Find the nature of the singularity of the function $\dfrac{1}{\sin^2 z}$ at $z = 0$.

10. Let u be a real valued piecewise continuous function on $[0, 2\pi]$. Define the Poisson integral of u.

11. Find the roots of the equation :

    $z^2 - 6z + 3 = 0$.

    in the annulus $\{z : 1 <$

12. Obtain the power series expansion of $\dfrac{1}{z}$ about $z = 1$ in the disk $\{z \mid |z-1| < 1\}$.

13. Prove that the sum of the residues of an elliptic function is zero.

14. Show that there does not exist an elliptic function with a single simple pole.

<div align="right">(14 x 1 = 14 weightage)</div>

## Part B

*Answer any* **seven** *questions.*

*Each question carries* 2 *weightage.*

15. Prove that a linear transformation carries circles into circle.

16. Describe the mapping properties of $W = e^z$.

17. State and prove Schwarz's lemma.

18. Let $r$ be a closed rectifiable curve. For any point 'a' not on $r$ define n $(r,$ a$)$. Show that n $(r, a)$ is always an integer.

19. Show that if $f(z)$ is analytic in a region $\Omega$ and statistics the inequality I f$(z)-1| < 1$ on $\Omega$, then :

$$\int_r \frac{f'(z)}{f(z)} dz = 0, \text{ for every closed curve } r \text{ in } \Omega.$$

20. Obtain the Laurent series expansion of $\dfrac{1}{z(z-1)}$ in :

    (i) $0 < |z| < 1$.     (ii) I $z| > 1$.

21. State and prove the Residue theorem.

22. If $f(z)$ is analytic in a region $\Omega$ and has no zeros in $\Omega$, prove that log $|f(z)|$ is harmonic in c.

23. Suppose $f$ has an isolated singularity at $z = a$. **If** $\lim\limits_{z \to a} (z-a) f(z) = 0$, show that $z = a$ is a removable singularity.

24. Show that any even elliptic functions with periods $w_1$ and $w_2$ can be expressed in the form :

$$C \prod_{k=1} \frac{p(z) - p(a_k)}{p(z) - p(b_k)}$$

where C is a constant.

<div align="right">(7 x 2 = 14 weightage)</div>

## Part C

*Answer any two questions.*
*Each question carries* 4 *weightage.*

25. State and prove Cauchy's theorem for a rectangle.

26. Using Residue theorem, evaluate the integral $\int_0^{-} \frac{}{a+\cos\theta}$ where a > 1.

27. Derive the formula for the Weierstrass elliptic function in the form :

$$p(z) = \frac{1}{z} + \sum_{w \neq 0}' \frac{1}{(z-w)}$$

28. Derive the Poisson integral formula for harmonic functions.

(2 x 4 = 8 weightage)