D 51671   (Pages : 3)   Name...

Reg. No

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2013

(CUCSS)

Mathematics

MT 3C 13—TOPOLOGY—II

Time : Three Hours   Maximum : 36 Weightage

Part A

*Answer **all** questions.*
Each question has weightage 1.

1. Define uniform convergence and give an example of it.

2. Let A be a subset of a topological space X and let $f : A \to \mathbb{R}$ be continuous. Prove that any *two* extensions of f to X agree on $\overline{A}$.

3. Prove that intersection of any families of boxes is a box.

4. Prove that if X_1 is a T_1–space for each $i \in I$ then $\prod_{i \in I} X_i$ is also a T_1–space in the product topology.

5. Let $X_i, i \in I$ be an indexed family of topological spaces and let $X = \prod_E X_i$. If $\{x_n\}$ is a sequence in X such that $\{x_n\}$ converges to $x \in X$, then prove that the sequence $\{\pi_i(x_n)\}$ converges to $\pi_i(x)$ in X_i for each i.

6. State Urysohn's metrization theorem.

7. Let X, Y and Z be topological spaces. If $h, h' : X \to Y$ are homotopic and $k, k' : Y \to Z$ are homotopic, then prove that $k \circ h$ and $k' \circ h'$ are homotopic.

8. Prove that the fundamental group of unit ball in $\mathbb{R}$ is trivial.

9. Define covering map and give an example of it.

10. Give an example of a countably compact space.

11. Are compact spaces sequentially compact ? Justify your answer.

12. Let A be a subset of a complete metric space (X ; d) such that A is complete w.r.to the metric induced on it. Prove that A is closed in X.

Turn over

13. Prove that finite union of totally bounded sets is totally bounded.

14. Prove that $\mathbb{R}$ with usual topology is of second category.

<div align="right">(14 x 1 = 14 weightage)</div>

<div align="center">

Part B

Answer any seven *questions.*
Each question has weightage 2.

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15. Let $\sum_{n=1} M_n$ be a convergent series of non-negative real numbers. Let $\{f_n\}$ be a sequence of real valued functions on a topological space X such that for each $x \in X$ and $n \in N$, $|f_n(x)| \leq M_n$. Prove that the series $\sum_{n=1} f_n$ converges uniformly to a real valued function on X.

16. Prove that projection functions are open.

17. Prove that a product of topological spaces is completely regular if and only if each co-ordinate space is completely regular.

18. Let X be a topological space and let $x \in X$. Let e_x be the constant path $e_x : I \to X$ carrying all of I to the point x. If f is a path in X from x_0 to x_1, then prove that $[f] * [e_{x_1}] = [f]$.

19. Prove that every continuous, real-valued function on a countably compact space is bounded and attains its extrema.

20. Prove that a subspace of a locally compact Hausdorff space is locally compact if and only if it is open in its closure.

21. Prove that every compact metric space is complete.

22. If a topological space X is regular and locally compact at a point $x \in X$, then prove that x has a local base consisting of compact neighbourhoods.

23. Show by an example that total boundedness is not topologically invariant.

24. Let $(x_1, d_1), (x_2, d_2)$ be complete metric spaces prove that $x_1 \times x_2$ is complete with respect to the metric :

$$d\ (\ x2), (h, y2) = \max \{d_1(xi \quad d_2(x_2, y_2)\}.$$

<div align="right">(7 x 2 = 14 weightage)</div>

Part C

Answer any two questions.
Each question has weightage 4.

25. Prove that a product of topological spaces is connected if and only if each co-ordinate space is connected.

26. Prove that a topological space is completely regular if and only if the family of all continuous real-valued functions on it distinguishes points from closed sets.

27. Prove that the map $p : \mathbb{R} \to S^1$ given by the equation :

$$p(x) = (\cos 2\pi x, \sin 2n x)$$

is a covering map.

28. Prove that any continuous function from a Tychnoff space into a compact, Hausdorff space can be extended continuously over the stone Cech compactification of the domain.

(2 x 4 = 8 weightag.