D 31327 (Pages : 3) **Name**

Reg. No. . . . .

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2012

(CUCSS)

Mathematics

MT 3C 13—TOPOLOGY—II

(2010 Admissions)

Time : Three Hours **Maximum : 36 Weightage**

Part A

Answer **all** *questions.*
Each, question has weightage 1.

1. Prove that the intersection of any family of boxes is a box.
2. Prove that the projection functions are open.
3. Prove that if; is a T_2-Space far each $i \in I$, then $\pi_{i \in I} X_i$ is also a T_2-space in the product topology.
4. Explain countably productive topological property.
5. Let $\{Y_i : i \in I\}$ be a family of sets, X a set and for each $i \in I$, Y_i a function. Prove that the evaluation function is the only function from X into πY_i whose composition with the projection π_i Y_i equals f_i for all $i \in I$.
6. State a necessary and sufficient condition for embedding a topological space in the Hilbert cube.
7. Define a simply connected space and *give* an example.
8. Show that if P : E B is a covering map, then *p* is an open map.
9. Let a covering map P : R S' be given by *p (x)* = (cos 2π *x*, sin 2n x). Find a lift of the path : [0,1] S' defined by *f (x)* = (cos πx, sin ax).
10. Prove that a continuous real-valued function on a countably compact space is bounded and attains its extrema.
11. Let $\mathcal{B}$ be a base for a topological space X such that every cover of X by members of 3 has a finite subcover. Prove that X is compact.

Turn over

12. Prove that every locally compact Hausdorff space is regular.

13. Give an example of a metric space which is complete but not compact.

14. Define Completion of a metric space (X, *d*).

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question has weightage 2.

15. Let A be a closed subset of a normal space X and suppose $f : A \to (-1,1)$ is continuous. Prove that there exists a- continuous function F: X (-1,1) such that F $(x) = f(x)$ for all x c A.

16. Let $\{(X_i, \tau_i) : i$ E I} be an indexed collection of topological spaces and let τ be the product topology on the set $\prod_{i \in I} X_i$ Prove that $\{$ (V$_1$): V$_1$ E τ_i, i E I $\}$ is a sub-base for T.

17. Let (X, *d*) be a metric space and let X. be any positive real number. Prove that there exists a metric *e* on X such that *e* (x, y) λ for all x, y E X.

18. Prove that a topological space is a Tychonoff space iff it is embeddable into a cube.

19. Prove that path homotopy —*p*) is an equivalence relation on the set of paths in a topological space X.

20. If h: (X, x_0) → (Y, y_0) is a homeomorphism of X with Y, prove that h_* *is* an isomorphism of π_1 (X, x0) with π_1 (Y, y_0).

21. Prove that sequential compactness is preserved under continuous functions.

22. Let X be a Tychonoff space, (e, (x)) its Stone-Cech compactification and suppose $f : X \to [0,1]$ is continuous. Prove that there exists-a map $g : \beta$ (X) → [0,11 such that goe = f.

23. Let A be a subset of a metric space (X, *d*) such that A is complete with respect to the metric induced on it. Prove that A is closed in X.

24. Prove that the range of the embedding $h : (X, d)$ $(\hat{X}, e)$ is a dense subset of $\hat{X}$ in the metric topology induced by e.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question has weightage 4.

25. **Prove that metrisability is a countably productive property.**

26. **Prove that a topological space is embeddable in the Hilbert cube iff it is second countable and T_3.**

27. **Let $p : E \to B$ be a covering map and let $p(e_0) = b_0$. Let the map F:1 x I $\to$ B be continuous with F (0, 0) = b_0. Prove that there is a lifting of F to a continuous map such that (0, 0) = e_0.**

28. **State and prove Alexander sub-base theorem.**

(2 x 4 = 8 weightage)