

D6718

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Name.....

Reg. No.....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 3C 14—LINEAR PROGRAMMING AND ITS APPLICATIONS

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** the questions.

Each question carries *weightage* 1.

1. Define boundary point of a set. Give an example of a boundary point of a set in E_3 , the three dimensional Euclidean space.
2. Define closed set and open set. Give an example of a set that is both open and closed.
3. Define the line in E_3 passing through two points X_1 and X_2 .
4. Prove that the convex hull of the set S is the set of all convex linear combinations of the points of S.
5. Define directional derivative of $f(X)$ in the direction of Y.
6. Distinguish between local extrema and global extrema.
7. Define **Lagrangian** function and Lagrange multipliers.
8. What is meant by loops in a transportation array ?
9. What is Caterer problem ?
10. When do we say that the transportation problem reduces to an assignment problem ?
11. Describe the general form of an integer linear programming problem in two dimensional space.
12. Define mixed integer vector.
13. Define pay off in a game. Give an example.
14. Define the terms saddle point and value of the game in theory of games.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** questions.

Each question carries *weightage* 2.

15. Prove that union of two open sets is open.
16. Give an example of a convex set with one vertex only.

Turn over

17. Find the convex hull of the set $S = \{(0,0,0), (1,0,0), (0,1,0), (0,0,1)\}$ in E_3 .
18. Show that if a polytope has a vertex, then it has an edge.
19. Prove that $f(x) = x^2$ is a convex function.
20. Define the dual of a linear programming problem. Prove that dual of the dual is the primal problem.
21. If the primal problem is feasible, prove that it has an unbounded optimum if and only if the dual has no feasible solution.
22. Describe the concept of loop in a transportation array.
23. By the cutting plane method :

$$\text{Minimise : } 4x_1 + 5x_2$$

$$\text{subject to : } 3x_1 + x_2 \leq 2$$

$$4x_2 \leq 5$$

$$3x_1 + 2x_2 \leq 7$$

x_1, x_2 non-negative integers.

24. Explain the terms mixed strategy, pure strategy and optimal strategies with reference to any matrix game.

(7 x 2 = 14 weightage)

Part C

Answer any **two** questions.
Each question carries **weightage** 4.

25. Use simplex method to :

$$\text{Maximize : } 3x_1 + 2x_2 + 3x_3$$

subject to the constraints :

$$x_1 + x_2 + x_3 \leq 9$$

$$2x_1 + 3x_2 + 5x_3 \leq 30$$

$$2x_1 - x_2 - x_3 \leq 8$$

$$x_1, x_2, x_3 \geq 0.$$

26. Solve that the transportation problem for minimum cost with the cost coefficients, demands and supplies as given in the following table. Obtain three optimal solutions.

	D_1	D_2	D_3	D_4	
O_1	1	2	-2	3	70
	2	4	0	1	38
O_3	1	2	-2		32
	40	28	30	42	

27. Solve the following integer linear programming problem :

$$\text{Maximize : } \phi(X) = 3x_1 + 4x_2$$

$$\text{subject to : } 2x_1 + 4x_2 \leq 13$$

$$-2x_1 + x_2 \geq 2$$

$$2x_1 + 2x_2 \leq 1$$

$$6x_1 - 4x_2 \leq 15$$

$$x_1, x_2 \geq 0$$

x_1 and x_2 are integers.

28. Use the notion of dominance to simplify the following payoff matrix and then solve the game:-

$$\begin{pmatrix} 10 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{pmatrix}$$

(2 x 4 = 8 weightage)