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Name.....

Reg. No.....

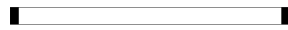
**THIRD SEMESTER M.Sc. DEGREE EXAMINATION DECEMBER 2015**

(CUCSS)

Mathematics

**MT 3C 14—LINEAR PROGRAMMING AND ITS APPLICATIONS**

Time : Three Hours



Maximum : 36 Weightage

**Part A (Short. Answer Type)**

Answer **all** the questions.  
Each question carries a *weightage* of 1.

1. Define convex set. Give an example for a convex set.
2. Prove that intersection of two convex sets is a convex set.
3. Prove that a **Hyperplane** is a convex set.
4. Is the function  $f(x) = x^2, x \in \mathbb{R}$ , a convex function. **Justify** your answer.
5. Distinguish between local and global extrema.
6. Define **Lagrangian** function and Lagrange multipliers.
7. Write the dual of the problem :

$$\text{Minimize } z = x_1 + 3x_2 \text{ subject to } x_1 + x_2 \geq 3, -x_1 + x_2 \leq 2, x_1 \geq 2, x_2 \geq 0, x_1 \geq 0, x_2 \geq 0.$$

8. What is meant by loops in a transportation array ?
9. What is meant by unbalanced transportation problem ?
10. Describe the **0--1** variable problems in integer programming.
11. Define artificial variables. Describe the uses of artificial variables in solving linear programming problems.
12. Describe the concept of primal and dual problems in optimization theory.
13. Describe matrix games.
14. Describe the notion of dominance in game theory.

(14 x 1 = 14 weightage)

Turn over

**Part B (Paragraph Type)**

Answer any **seven** questions.

Each question carries a *weightage* of 2.

15. If  $S_F$  denote the set of feasible solutions of a general linear programming problem, then prove that a vertex of  $S_F$  is a basic feasible solution.
16. Use the method of Lagrange multipliers to find the maxima and minima of  $x_2^2 - (x_1 + 1)^2$  subject to  $x_1 + x_2 = 1$ .
17. Find the relative maxima and minima and saddle points if any of:  $f(x) = x_1 + x_2 - 3x_1 - 12x_2 + 25$ .
18. Define the dual of a linear programming problem. Prove that if the primal problem is feasible, then it has an unbounded optimum if and only if the dual has no feasible solution, and vice versa.
19. Find the point in the plane  $x_1 + 2x_2 + 3x_3 = 1$  in  $E_3$  which is nearest to the point  $(-1, 0, 1)$ .
20. Discuss degeneracy in transportation problems.
21. Prove that the transportation problem has a triangular basis.
22. Describe the rectangular game as a Linear programming problem.
23. Write the general form of an integer linear programming problem.
24. Explain the terms mixed strategy, pure strategy and optimal strategies with reference to any matrix game.

(7 x 2 = 14 weightage)

**Part C (Essay Type)**

Answer any **two** questions.

Each question carries a *weightage* of 4.

25. Use simplex method to solve the problem :

Maximize  $f(X) = 5x_1 + 3x_2 + x_3$  subject to the constraints

$$2x_1 + x_2 + x_3 = 3, \quad -x_1 + 2x_3 = 4, \quad x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

26. Solve the transportation problem for minimum cost starting with the degenerate solution  $x_{12} = 30, x_{21} = 40, x_{32} = 20, x_{43} = 60$ .

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	
O <sub>1</sub>	4	5	2	30
O <sub>2</sub>	4	1	3	40
O <sub>3</sub>	3	6	2	20
O <sub>4</sub>	2	3	7	60
	40	50	60	

27. Solve the following integer linear programming problem :

Maximize  $\phi(X) = 3x_1 + 4x_2$  ; subject to  $2x_1 + 4x_2 \leq 13$ ,

$-2x_1 + x_2 \leq 2$ ,  $2x_1 + 2x_2 \leq 1$ ,  $6x_1 - 4x_2 \leq 15$ ,  $x_1, x_2 \geq 0$ ,  $x_1$  and  $x_2$  are integers.

28. Solve the game where the pay-off matrix is  $\begin{vmatrix} 2 & \\ 3 & 5 \\ 0 & 1 & 2 \end{vmatrix}$

(2 x 4 = 8 weightage)