D 51672 **(Pages : 3)** **Name..................................**

Reg. No...............................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2013

(CUCSS)

Mathematics

MT 3C 14—LINEAR PROGRAMMING AND ITS APPLICATIONS

Time : Three Hours **Maximum : 36 Weightage**

Part A

Answer **all** *questions from Part A.*
Each question has weightage 1.

1. **Prove that the set S = {X E E_n : | X I = 1} is not convex.**

2. **Find the convex hull of the set S in E_3 where S = {(1, 0, 0), (0,1, 0), (0, 0,1)}.**

3 **Find ∇f(X) and 11(X) for $f(X) = x_1^3 + 2x_3^3 + 3x_1 x_2 x_3 + x_3^2$.**

4. **Prove that the set of feasible solutions to a general LP problem is a convex set.**

5. **Write the following LP problem in standard form :**

Maximize Z = $2x_1 + 3x_2 + 5x_3$

subject to $x_1 + x_2 -$ —5

$-6x_1 + 7x_2 - 9x_3$ 4

x1 $+x2 + 4x_3$ 10

x_2 >0

x_3 **unrestricted in sign.**

6. **Define slack and surplus variables in an LP problem.**

7. **What is degeneracy in an LP problem ?**

8. **Prove that the dual of the dual of an LP problem is the primal problem.**

9. **What do you mean by loop in a transportation array ?**

10. **Show that the optimal solution of an assignment problem is unchanged if we add or substract the same constant to the entries of any row or column of the cost matrix.**

Turn over

11. What do you mean by mixed integer programming problem ?

12. Describe two-person zero-sum game.

13. State the fundamental theorem of rectangular games.

14. Solve the game whose pay-off matrix is :

$$\begin{bmatrix} -3 & -2 & 6 \\ 2 & 0 & 2 \\ 5 & -2 & -4 \end{bmatrix}$$

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions from Part* **B.**

Each question has weightage 2.

15. Give example of a convex set with :

(a) no vertex.

(b) one vertex only.

16. Let $f(X)$ be a convex differentiable function defined in a convex domain $K \subset E$. Prove that $f(X_0)$, $X_0 \in K$ is a global minimum iff :

$(X - X_0)' \nabla f(X_0)$ 0 for all X in K.

17. Prove that every positive linear combination of convex functions in K is a convex function in **K.**

18. Prove that a basic feasible solution of an LP problem is a vertex of the convex set of feasible solutions.

19. Prove that the optimum value of the primal, if it exists is equal to the optimum value of the dual.

20. Write the dual of the LP problem :

Maximize $Z = 4x_1 + 5x_2 + 3x_3$

subject to $4x_1 + x_3 \leq 420$

$2x_2 + 3x_3$ 460

$2x_1 + x_2 + x_3 \leq 500$

x_1, x_2, x_3 0.

Without carrying the simplex computations, estimate a range for the optimal value of the objective function.

21. Find a feasible solution of the following transportation problem :

	D_1	D_2	D_3	D_4	*Available*
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Required	20	40	30	10	100

22. Explain why the transportation algorithm is not appropriate for solving the assignment problem.

23. Define saddle point in a game. Is it necessary that a game should always possesses a saddle point.

24. Use the notion of dominance to simplify the following pay-off matrix and then solve the game :

$$\begin{matrix} 0 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{matrix}$$

(7 X 2 = 14 weightage)

Part C

Answer any two questions from Part C.
Each question has weightage 4.

25. Find the, maximum and minimum values of $|X|^2$, $X \in E_3$, subject to the constraints :

$$\frac{x_1^2}{4} + \frac{x_2^2}{5} + \frac{x_3^2}{25} = 1 \text{ and } x_1 + x_2 - x_3 = 0.$$

26. Solve the LP problem :

$$\begin{aligned} \text{Maximize } Z &= 3x_1 + 2x_2 + x_3 + 4x_4 \\ \text{subject to } & 2x_1 + 2x_2 + x_3 + 3x_4 \leq 20 \\ & 3x_1 + x_2 + 2x_3 + 2x_4 \leq 20 \\ & x_1, x_2, x_3, x_4 \geq 0. \end{aligned}$$

27. Describe a method of solving mixed integer programming problem.

28. Solve graphically the game whose pay-off matrix is given by :

$$\begin{bmatrix} 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix}.$$

(2 X 4 = 8 weightage)