D 51669 **(Pages : 3)** **Name**

Reg. No....................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2013

(CUCSS)

Mathematics

MT 3C 11—COMPLEX ANALYSIS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Find the fixed points of the linear transformation $w = \frac{2z}{3z-1}$

2. Find the points at which the function tan z is not analytic.

3. State the symmetry principle.

4. If $z = x + iy$, prove that $|e^z| = e^x$.

5. Compute $\int_r y\,dz$ where r is the directed line segment from 0 to i.

6. Let n be a positive integer. Prove that $\int_r (z-a)^n\,dz = 0$ for any closed curve r.

7. Let the curve r lie inside of a circle. Prove that the index n $(r, a) = 0$ for all points a outside of the same circle.

8. Determine the nature of the singularity of the function $\frac{\sin z}{\ }$ at $z = 0$. Justify your answer.

9. Find the residue of the function $f(z) = \frac{z^2 - 2}{(z-2)^2}$ at $z = 2$.

10. Define : Simply connected region. Give an example of a simply connected region.

11. Prove : the argument 0 is harmonic wherever it can be defined.

Turn over

12. Find a harmonic conjugate of the function $e^x \cos y$.

13. Find the Taylor series expansion of the function $\frac{1}{z^2}$ at $z = 1$.

14. Prove that an elliptic function without poles in constant.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries **2** *weightage.*

15. Let Q be the region C – {z : z e., the complement of the negative real axis. Define a continuous function $f : \Omega \to C$ satisfying $f(z) = z^2$ for all $z \in \Omega$ and f (1) = 1. Show that f is analytic in Q.

16. Define : Linear fractional transformation prove that a linear fractional transformation is a topological mapping of the extended complex plane into circles.

17. Prove that the cross-ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or a straight line.

18. Let f be a continuous complex valued function defined on the closed interval [a, b]. Prove that

$$\left| \int_a f(t)\, dt \right| \quad \int_a f(01\, dt.$$

19. State and prove Morera's theorem.

20. Prove that an analytic function comes arbitrarily close to any complex value in every neighbourhood of an essential singularity.

21. How many roots does the equation $z^7 - 2z^5 + 7z^3 - z + 1 = 0$ have in the disc $|z| < 1$.

22. State and prove Hurwitz's theorem.

23. Find the Laurent series expansion of the function $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions $0 < |z - 1| < 1$ and $1 < |z - 2| < \infty$.

24. Derive Legendre's relation $\eta_1 \omega_2 \quad = 2\pi i$.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries **4** weightage.

25. Let the function $f(z)$ be analytic on the rectangle R defined by the inequalities

Then prove that $\int_{\partial R} f(z)\, dz = 0.$

26. Let the function f be analytic in a region Ω and let $a \in \Omega$. Suppose that $f(a)$ and all its derivatives $f^{(n)}(a)$ vanish. Prove that f is identically zero in Ω.

27. Discuss the evaluation of integrals of the type $\int R(x)\, e^{ix}\, dx$ using the theory of residues.

28. Prove that the Weirestrass eliptic function satisfies the differential equation of the form

$$(\wp'(z))^2 = 4(\wp(z))^3 - g_2\, \wp(z) - g_3.$$

(2 x 4 = 8 weightage)