C 63077 (Pages : 3) Name..............................

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SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2019

(CUCSS)

Mathematics

MT 2C 10—ODE AND CALCULUS OF VARIATIONS

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Each question carries 1 weightage.

1. Find a power series solution of the form $E a_n x''$ of the equation $y' = (1-x^2)^{-\frac{1}{2}}$.

2. Determine the nature of the point x = 0 for the equation $x^3 y'' + (\sin x)y = \mathbf{0}$.

3. Show that sin x = x[lim F a, a, $\frac{3}{2}$, $-\frac{x^2}{a^2}$] (a->x) r 2 1.

4. Transform the equation (1 − e x) y'' + ... e y = 0 into a hypergeometric equation.

5. Show that Pen (0) = 1.3...(2n -1) / 2^n . n! where $P_n(x)$ denotes the nth degree Legendre polynomial.

6. Prove that Γ(n + 1) = n! for any integer n 0.

7. Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

8. Describe the phase portrait of the system $\frac{dx}{dt} = 1, \frac{dy}{dt} = 2$.

9. Find the critical points of the system :

$$\frac{dx}{dt} = y(x^2 \quad = \frac{dy}{dt} = -x(x^2 + 1$$

10. Determine whether the function $-2x^2 + 3xy - y^2$ is positive definite, negative definite or neither.

11. State Picard's theorem.

Turn over

12. Show that f (x,y) = y^{112} satisfies a Lipschitz condition on the rectangle Ix I 1, 1 < y 2.

13. Find the stationary function of $\int_0$ x.Y' -(y 121 which is determined by the boundary conditions y(0) = 0, y(4) = 3.

14. Find the normal fbrm of Bessel's equation $x^2y'' + xy' + (x^2 - \text{——} y = 0$.

(14 x 1 = 14 weightage)

Part B

Answer any seven *questions.*
Each question carries 2 *weightage.*

15. Find the general solution of the equation $(1 + x^2)y'' + 2xy' - 2y = 0$.

16. Find the indicial equation and its roots of the equation $x^3 y'' + (\cos 2x - 1)y' + 2xy = 0$,

17. Show that the solutions of the equation $(1 - x^2 y'' - 2xy' + n(n+1)y = 0$, where n is a non-negative integer, bounded near $x = 1$ are precisely constant multiples of $F\left\| -n, n+1, 1, \frac{1}{2} - \right|$

18. Obtain the Bessel function of the first kind $J_p(x)$.

19. Prove that the positive zeros of $J_p(x)$ and $J_{p+i}(x)$ occur alternately, in the sense that between each pair of consecutive positive zeros of either there is exactly one zero of the other.

20. Determine the nature of stability properties of the critical point (0, 0) for the system :

$$\frac{dx}{dt} = 5x + 2y, \frac{dy}{dt} = -17x - 5y.$$

21. Show that (0, 0) is au asymptotically stable critical point of the system :

$$\frac{dx}{dt} = y\ x^3 \frac{dy}{dt} = x - y^3.$$

22. Let $u(x)$ be any non-trivial solution of $u'' + q(x)u = 0$, where $q(x) > 0$ for all $x > 0$. Show that if $\int_0 q(x)dx = oo,$ then u(x) has infinitely many zeros on the positive x-axis.

23. Find the exact solution of the initial value problem $y' = y^2, y(0) = 1$. Starting with $y_0(x) = 1$, apply Picard's method to calculate $y_1(x)$, $y_2(x)$, $y_3(x)$, and compare these results with the exact solution.

24. Using the method of Lagrange's multipliers, find the point on the plane $ax + by + cz = d$ that is nearest the origin.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries 4 weigh.tage.

25. Calculate two independent Frobenius series solutions of the equation $2x^2y'' + xy' - (x + 1)y - 0$.

26. Solve the hypergeometric equation $x(1-x)y'' + [c - (a + b + 1)x] y' aby = 0$, near its singular point $x = 0$,

- 27. Find the general solution of the system :

$$\_\frac{dx}{dt} \quad 4x - y, \quad \frac{}{at} = x - 2y.$$

28. Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition $(x, y_i) - f(x, y_2)$ - y21 on a strip defined by a x < b and - a < y < oo. If (x_0, y_0) is any point of the strip, then show that the initial value problem $y' = f(x, y), y(x_0) = y_o$ has one and only one solution y $f(x)$ on the interval $a < x < b$.

(2 x 4 = 8 weight:age)