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Name................................

Reg. No..............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016

(CUCSS)

Mathematics

MT 2C 10—NUMBER THEORY

(2010 Admissions)

Time : Three Hours — Maximum : 36 Weightage

Part A

Answer all questions.
Each question has weightage of 1.

1. If n 1, prove that $\sum_{din}$ *(d)* n.

2. Define Dirichlet product of two arithmetical functions. If f is an arithmetical function, find I such that $f * I = I * f = f$

3. Define a multiplicative function. If f is multiplicative, prove that f^{-1} is multiplicative.

4. If f and g are arithmetical functions, prove that $(f * g)' = f' * g + f * g'$.

5. With usual notations, prove that $\Lambda * u = u'$ and hence derive the Selberg identity.

6. Prove that $[2x] + [2y]$ $[x] + [y] + [x + y]$.

7. Let $f(x) = x^2 + x + 41$. Find the smallest integer for which $f(x)$ is composite.

8. Solve the congruence $5x \equiv 3 \pmod{24}$.

9. Find the quadratic residues and non-residues modulo **11.**

10. State and prove Little Fermat's theorem.

11. Determine whether -104 is a quadratic residue or quadratic non-residue of 997.

12. Define an affine crypto-system. Illustrate with an example.

Turn over

13. Prove that product of two linear enciphering transformation is a linear enciphering transformation.

14. Describe how a signature is sent in RSA.

(14 x 1 = 14 weightage)

Part B

Answer any seven *questions.*
Each question has weightage 2.

15. If n 1, prove that A (n) = $\sum_{d/n}$ *(d)* $\log\left(\frac{n}{d}\right)$ – $\sum_{d/n}$ *(d)* log *d.*

16. Let f be a multiplicative function. Show that f is completely multiplicative if and only if f^{-1} (n) = *(n)* (n) for n **1** .

17. State and prove the Euler's summation formula.

18. If a and *b* are positive real numbers such that $ab = x$ and land g are arithmetical functions with

F (x) = $\sum_{n \le x}$ f (n) and G *(x)* $\sum_{n \,.\, x}$ *g (n),* prove that

f *(d) g (q)*= $\sum_{n \le a}$ $f(n)\,G$ + $\sum$ $g(n)\,F$ F (a) G *(b).*

19. With usual notations, prove that, for x > 0, $\frac{(\log x)^2}{2\sqrt{x}\log 2}$ — $\frac{\psi(x)}{x}$ — $\vartheta(x)$

20. For any prime p 5, prove that $\sum_{k=1}$ $\frac{(p-1)!}{k}$ 0 (mod p^2)

21. Prove that the Legendre's symbol (n I *p)* is a completely multiplicative function of n.

22. If p and q are distinct odd primes, prove that *(p q) (q p)* = $(-1)^{\frac{(p-1)(q-}{4}}$

23. In the 27- letter alphabet with 'blank = 26', $A = \begin{pmatrix} 2 & 3 \\ 7 & 8 \end{pmatrix} \in$ **M2 (Z/26 Z),** to encipher the message unit "NO" assuming each plaintext message unit $P = \begin{pmatrix} x \\ y \end{pmatrix}$ is transformed into $C = \begin{pmatrix} x' \\ y' \end{pmatrix}$ by the rule $C = A P$.

24. Solve the congruences :

$$x + 4y \equiv 0 \pmod 9$$

$$5x + 8y \equiv 0 \pmod 9$$

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question has weightage of 4.

25. Show that $\frac{1}{6} \frac{n}{\log n} < \pi(n) < \frac{6n}{\log n}$, for every integer $n \geq 2$.

26. With usual notations, prove that there is a constant A such that

$$\sum_{p \leq x} \frac{1}{p} = \log(\log x) + A + O\left(\frac{1}{\log x}\right) \text{ for all } x \geq 2.$$

27. (a) Find all odd primes for which 3 is a quadratic residue.

(b) Find all odd primes for which 2 is a quadratic non-residue.

28. Describe algorithm for finding the discrete logs in the finite fields.

(2 x 4 = 8 weightage)