C 44044 **(Pages :** **Name..**

Reg. No..

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, AUGUST 2013

(CUCSS)

Mathematics

MT 2C 08—TOPOLOGY—I

Time : Three Hours **Maximum : 36** Weightage

Part A (Short Answer Type Questions)

Answer all the questions

(Each question has Weightage One)

1. Define open set in a metric space. Give an example of an open set.
2. Define discrete and in-discrete topologies on a set.
3. Give an example of an open set which is not an open interval in the set of real numbers with usual topology. Justify your claim.
4. Define connected sets in topological spaces. Give an example for a connected set.
5. Define diameter of a set in a metric space. Illustrate using an example.
6. Define homeomorphism from one topological space to another. Give an example.
7. Distinguish between path connectedness and connectedness in topological spaces,
8. Give an example of a topological space that is T_0 but not T_1.
9. Define embedding of a topological space into another.
10. Distinguish between open maps and closed maps in topological spaces.
11. Define weakly hereditary properties. Give **examples of any two weakly hereditary properties.**
12. **Give an example of a regular space which is not.** T3.
13. **State the Lebesgue covering lemma.**
14. **State the Tietze characterization of normality.**

(14 x 1 = 14 weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any seven questions

Each question has weightage **two**

15. Prove that the semi-open interval topology is stronger than the usual topology on the set of real integers.

16. If a space is second countable, prove that every open cover of it has a countable sub cover.

17. Define hereditary property in a topological space. Prove that second countability is a hereditary property.

18. If A and ***B*** are any two subsets of a topological space ***X*,** prove that

19. Prove that a subset A of a space X is dense in ***X*** if and only if for every non empty open set ***B*** of ***X*,** A n ***B*** ϕ.

20. Prove that the interior of a set is the same as the complement of the closure of the complement of the set.

21. Prove that inverse image of an open set under a continuous function is open.

22. Prove that a compact subset of a Hausdorff space is dosed.

23. Prove that every open surjective map is a quotient map.

24. Prove that every separable space satisfies the countable chain condition.

(7 x 2 = 14 weightage)

Part C (Essay Type Questions)

Answer any two questions

Each question has weightage four

25. (a) Prove that the usual topology in the euclidean plane $\mathcal{R}^2$ is strictly weaker than the topology induced on it by the lexicographic ordering.

(b) Let ***X*** be a set, r a topology on ***X*** and ***S*** a family of subsets on X. Then prove that ***S*** is a sub-base for ***r*** if and only if *S* generates r.

26. (a) Prove that metrisability is a hereditary property.

(b) Prove that composition of continuous functions is continuous.

27. (a) Prove that a discrete space is second countable if and only if the underlying set is countable.

(b) Prove that every closed and bounded interval is compact.

28. (a) Prove that every quotient space of a locally connected space is locally connected.

(b) State and prove Urysohn's lemma.

(2 x • = 8 weightage)