C 63076 **(Pages : 2)** **Name..............................**

Reg. No.............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2019

(CUCSS)

Mathematics

MT 2C 09—TOPOLOGY

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*

Each question carries a *weightage of 1.*

1. Define bounded set in a metric space. Write an example of a bounded set in the set of real numbers with usual metric.
2. Define convergent sequence in a metric space. Prove that constant sequence in any metric space is convergent.
3. Give examples of two topologies on a finite set that are not comparable.
4. Define semi open interval topology on the set of real numbers.
5. Define scattered line. Prove that every subset of irrational numbers is open in the scattered line.
6. Define accumulation point of a set. Give an example for accumulation point of a set in a topological space.
7. Define topological space in terms of closed sets.
8. Prove that the composite of two continuous functions is continuous in a topological space.
9. Define open map, surjective map and quotient map in a topological space. State a relation connecting the three.
10. What is meant by embedding problem in topological space ? Explain.
11. Prove that the property of being a finite space is divisible.
12. Define absolute property and relative property of a subset of a topological space.
13. Prove that a metric space is a T_1 space.
14. Define regular space, Lindeloff space and normal space. State a relation connecting the three.

(14 x 1 = 14 weightage)

Turn over

Part B

Answer any seven *questions.*
Each question carries a *weightage of* 2.

15. Prove that open balls in a metric space are open sets.
16. Determine the topology induced by a discrete metric on a set.
17. Define co-countable topology. Prove that in the co-countable topology the only convergent sequences are those which are eventually constant.
18. State the second axiom of countability. Prove that if a space is second countable, then every open cover of it has a countable subcover.
19. Define exterior of a set in a topological space. Prove that exterior of any set in a topological space is the complement of the closure of the set.
20. For any three spaces X_1, X_2, X_3 prove that X_1 x (X_2 x X_3) = (X_1 x X_2) x X_3.
21. Prove that product topology is the weak topology determined by the projection functions.
22. Prove that compactness is weakly hereditary property.
23. Prove that the topological product of any finite number of connected space is connected.
24. If a space X has the property that for any two mutually disjoint closed subsets A and B of it, there exists a continuous function *f : X* -> [0, 11 taking the value at all points of A and the value 1 at all points of **B,** then prove that Xis normal.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries a *weightage of* 4.

25. (a) Let *X* be a set, t a topology on *X* and *S* a family of subsets on *X*. Then prove that *S* is a sub-base for t if and only if *S* generates T.

    (b) Prove that a subset A of a space *X* is dense in *X* if and only if' for every non-empty open subset B of *X, A* n *B# 0.*

26. (a) For any subset A of a space X, with usual notations prove that A = *A u*

    (b) Prove that the metric topology on IR` is the same as the product topology on a^2

27, (a) Define nearness relation on a set. Prove that there is a one-to-one correspondence between the set of topologies on a set and the set of ail nearness elations on that set

    (b) Prove that every continuous real-valued function on a compact space is bounded u :1 attains its extrema.

28. (a) Prove that a subset of the set of real numbers with usual topology is connected if and only if it is an interval.

    (b) Let A be a closed subset of a normal space *X* and suppose *f : A --> [-1, 1.]* is a continuous function. Then prove that there exists a continuous function *F X* [4,11 such that *F (x) = f (x)* for all X E A.

(2 x 4 = 8 weightage)