C 4668

(Pages : 3)

Name..............................

Reg. No...........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2016

(CUCSS)

Mathematics

MT 2C 06—ALGEBRA II

(2010 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. **Is** the ring z x z an integral domain. Justify your answer

2. Let *p (x)* be an irreducible polynomial of degree > 1 in $F[x]$ and let $I = \langle p(x) \rangle$. Show that $a + I \neq b + I$ for $a \neq b$ in F.

3. Show that Q ( ) is an algebraic extension of Q.

4, **Find the degree [Q** (a) : Q] where a = +

5. Find **the degree of c over R where c is the field of complex numbers and** $\mathbb{R}$ **is the field of** reals.

6. **Let** a **be a real number such that [Q (a) : Q] = 4. Is** a constructible. **Justify your answer.**

7. **Let** a be a zero of $x^2 + x + 1 \in \mathbb{Z}_2$ *[x]* and let F = Z2 (a). List all the elements of F.

8. Let $a : Q(\sqrt{2}) \to Q$ (5) **be defined by a (a + b** $b + a$ *f* where *a, b* $\in$ Q. verify whether a is an automorphism of $Q(\sqrt{2})$.

9. **Let E be the splitting field of** x^3 **—1 over** Q. Find [E : Q

10. **Find** the index {Q (a) : Q} where a =

11. List the elements of the Galois group G (Q (1 + *i*) / Q).

Turn over

12. Verify whether $(y_1 - 1)(y_2 - 1)(y_3 - 1)$ is a symmetric function in y_1, y_2, y_3.

13. Describe the third cyclotomic, polynomial $\Phi_3(x)$ over Q.

14. Verify whether $x^5 - 2$ is solvable by radicals over Q.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carris weightage 2.

15. Let N be an ideal in a commutative ring R and let $a \in R$.. Show that $I = \{ra + n : r \in R, n \in N\}$ is an ideal of R containing N.

16. Let E be an extension of a field F and $a \in E$. Let $p(x)$ be an irreducible polynomial in $F[x]$ such that $p(a) = 0$. Show that if $f(x) \in F[x]$ is such that $f(a) = 0$ then $p(x) \mid f(x)$.

17. Let E be an extension of a field F, $a \in E$ and let $\phi_a : F[x] \to E$ be the evaluation homomorphism. Show that a is transcendental over F if and only if ϕ_a is one-to-one.

18. Let E be an extension of a field F and let $K = \{a \in E : a$ is algebraic over $F\}$. Show that K is a subfield of E.

19. Show that every finite extension of a finite field is a simple extension.

20. Let E be an extension of a field F and $a \in E$ be algebraic over F. Let σ be an automorphism of E leaving F fixed. Show that $\sigma(a)$ is a zero of $irr(a; F)$.

21. Let K be the splitting field of $x^3 - 2$ over Q. Find $[K : Q]$.

22. Describe all elements of the Galois group G (K/ Q) where K is the splitting field of $x^3 + 2$ over Q.

23. Let **H** be a subgroup of a Galois group G(K/F). Show that $K_H = \{a \in K : \sigma(a) = a$ for all $\sigma \in$ **H**$\}$ is a subfield of K.

24. Show that a regular 7-gon is not constructible by straight edge and compass.

(7 x 2 = 14 weightage)

Part C

· *Answer any two questions.*
Each question carries weightage 4.

25. Let F be a field. Show that every ideal in $F[x]$ is a principal ideal. Let $p(x)$ be irreducible in $F[x]$. Show that $\langle p(x) \rangle$ is a maximal ideal in $F[x]$, Verify whether $x^3 + x^2$ 2 is irreducible in Z3 [x].

26. Define algebraically closed field. Show that a field F is algebraically closed if and only if every non constant polynomial in $F[x]$ factors into linear factors in $F[x]$.

27. Define splitting field. Let E, F be fields such that $F \leq E \leq \overline{F}$. Show that E is a splitting field over F if and only if every isomorphism from E into F leaving $\overline{F}$ fixed maps E onto E.

28. Describe the 8th cyclotomic polynomial $\Phi_8(x)$ over Q.. Show that $\Phi_8(x) = x^4 + 1$. Describe the. Galois group of Φ (x) over Q.

(2 x 4 = 8 weightage)