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Name...

Reg. No

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA—I

(2016 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Verify whether +(x, y) = (x + y, 0) is an isometry of the plane.
2. Find the order of (2, 6) in the group Z_4 x Z_{12}.
3. Give two non-isomorphic groups of order 8.
4. Let G be the cyclic group Z_4 and X = {1, 2, 3, 4} with action given by $a \cdot x = a + x$ (mod 4). Find the isotropy group G_x for $x = 1$.
5. Verify whether the series (0) < < 5 > < Z15 and (0) < < 3 > < 76$_{15}$ are isomorphic.
6. Find the commutator subgroup of the symmetric group 8_3.
7. Find a subgroup of order 4 in Z_6 x Z_6.
8. Find the number of 3-sylow subgroup of a group G where I G1=18.
9. Let II, K be subgroups of a group G and H n K = {e}. Show that if $h_1k_1 = h_2k_2$ for some h_1, h_2 E H and $k_1, k_2 \in K$ then $h_1 = h_2$ and $k_1 = k_2$.
10. Find the number of elements in the group presented as $\langle x, y : xy = x, x^2 y = y \rangle$.
11. Let : $Q[x]$ Q be the evaluation homomorphism at 2. Find Ker ϕ.
12. Verify whether $x^2 - x$ is irreducible in. $Q[x]$.
13. Find the inverse of (1 + 2i + 2j) in the ring of quaternions.
14. Verify whether N = {0, 2, 4) is an ideal of the ring Z_6.

(14 x 1 = 14 weightage)

Turn over

Part B

Answer any seven *questions.*
Each question has weightage 2.

15. Find all generators of the cyclic group $7L_3$ x Z_4.

16. Show that there are only two non-isomorphic groups of order 25.

17. Let G = Z_4 x Z_6. Find a subgroup H of order 2 in G such that Gill is cyclic.

18. Let G be the symmetric group S_4 and X = {1, 2, 3, 41 with action given by a • x = *(x)* for all a ∈ G and x ∈ X. Find the number of orbits in X.

19. Let N be a normal subgroup of a group G and H be a subgroup of G. Show that HN = NH.

20. Show that S_3 is a solvable group.

21. Show that a free group on one generator is isomorphic of (Z, +).

22. Show that the group presented by $\langle x, y : x^2 = y^3 = 1, xy = yx \rangle$ is isomorphic to the cyclic group Z_6.

23. Verify whether $x^5 - 3x^3 + 9x + 6$ is irreducible in $\mathbb{Z}[x]$.

24. Let N be an ideal of a ring R. Show that 4) : R → R/N defined by $x \mapsto x + N$ is a homomorphism of rings.

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question has weightage 4.

25. (a) Show that $\mathbb{Z}_m$ x $\mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$ if and only if gcd of m and n is 1.

    (b) Show that if a is of order m in a group G_1 and ***b*** is of order n in a group $\mathbf{G}_2$ then the order of *(a, b)* in G_1 x G_2 is the */cm* of m and n.

26. (a) Define simple group.

    (b) Let G be a group and M be a normal subgroup of G. Show that G/M is simple if and only if M is a maximal normal subgroup of G.

27. Let H be a subgroup of G and N be a normal subgroup of G. Show that :

    (a) N is a normal subgroup of HN.

    (b) H n N is a normal subgroup of H.

    (c) HN/N is isomorphic to H/(H n N).

28. (a) State the division algorithm in F[x] where F is a field.

    (b) Show that the quotient and the remainder are unique in the division.

    (c) Show that if a E F is a zero of $f(x)$ e F $[x]$ then $(x - a)$ is a factor of $f(x)$.

(2 x 4 = 8 weightage)