

D 13182

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Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 01—ALGEBRA—I

(2010 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** questions.

Each question carries **weightage** 1.

1. Verify whether the map $(x, y) \mapsto (x+1, y)$ is an isometry of the plane which maps the X-axis to X-axis.
2. Verify whether the direct product $Z_2 \times Z_4$ is isomorphic to Z_8 .
• Find the order of the element $(2, 3)$ in $Z_3 \times Z_6$.
4. For binary codes u, v of same length prove that if $d(u, v) = 0$ then $u = v$.
5. List the elements of the quotient group Z_6/H where H is the subgroup generated by 2.
6. Give a composition series which is a refinement of $(0) \subset \langle 3 \rangle \subset 5 \subset Z_{12}$.
7. Let H be a subgroup of a group G and G be an H -set defined by $h * g = hg$. Find the orbit of g where g is an element of G and $g \in H$.
8. Find the number of **Sylow** 5-subgroups of a group of order 15.
9. If $w = a_1 a_2^{-2} a_3^{-1}$ and $v = a_3 a_2^{-1} a_1$ find the reduced word corresponding to wv .
- 10. List all the elements of the group whose presentation is $(x, y : x^2 y^{-1} = xy, xy^{-1} x = y)$.
11. Verify whether $x - 1$ is a factor of $2x^3 - 3x^2 + 1$ in $\mathbb{Q}[x]$.

Turn over

12. Verify whether $x^5 + 3x^3 + 6x + 3$ is irreducible in $\mathbb{Q}[x]$.
13. Find the multiplicative inverse of $2i + j + k$ in the skew field of quaternions.
14. Verify whether the ring of all 2×2 matrices over \mathbb{R} is a field.

(14 x 1 = 14 weightage)

Part B*Answer any seven questions.**Each question carries weightage 2.*

15. Prove that $Z_3 \times L_5$ is isomorphic to 76_{15} .
16. Prove that if m divides the order of a finite abelian group G then G has a subgroup of order m .
17. Let H be a normal subgroup of a group G and $\phi: G \rightarrow G/H$ be defined by $\phi(x) = xH$. Show that $\ker \phi = H$.
18. Let $G = H \times K$ be a direct product of groups. Let $\bar{K} = \{(e, k) : k \in K\}$. Show that G/\bar{K} is isomorphic to H .
19. Let X be a G -set. Let \sim be a relation on X defined by $x \sim y$ if $y = gx$ for some $g \in G$. Show that \sim is an equivalence relation on X .
20. Let H and K be normal subgroups of a group G such that $K \subset H$. Show that G/K is isomorphic to $(G/H)/(H/K)$.
21. Prove that every group of order 81 is solvable.
22. Let $F[x]$ be ring of polynomials over a field F . Let $\phi: F[x] \rightarrow F$ be defined by $a_0 + a_1x + \dots + a_nx^n \mapsto a_0$. Show that ϕ is a homomorphism of rings.
23. Show that $x^3 = 2$ has no solutions in rational numbers.
24. Show that quaternion multiplication is not commutative.

(7 x 2 = 14 weightage)

Part C

Answer any **two** questions.

Each question carries *weightage* 4.

25. Define direct product of two groups G_1 and G_2 . Prove that if $a \in G_1$ is of order m and $b \in G_2$ is of order n then order of $(a, b) \in G_1 \times G_2$ is the **lcm** of m and n .
26. Let X be a G -set. For each $g \in G$ let $\sigma_g : X \rightarrow X$ be defined by $x \mapsto gx$. Prove that σ_g is a permutation on X .
27. Define **Sylow** p -subgroups of a group G . Show that any two **Sylow** p -subgroups of a group G are conjugates.
28. Show that the set of all **quaternions** form a skew field, but not a field.

(2 x 4 = 8 *weightage*)