D 52971 **(Pages : 2)** Name....................................

Reg. No.................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014

(CUCSS)

Mathematics

MT IC **01—ALGEBRA—I**

Time : Three Hours Maximum : 36 **Weightage**

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Define an isometry of R^2 and show that the product of two **isometries** is again an isometry.
2. Find the order of the element (3, 10, 9) in $Z_4 \times Z_{12} \times Z_{15}$.
3. Show that for word addition of binary words u and v of the same length, we have u + v = u − v.
4. Let H and K be subgroups of a group G. Give an example showing that we may have H = K while G/H is not isomorphic to G/K.
5. Define solvable group and give one example of it.
6. Let G be a group of order p^n and let X be a finite G-set. Show that $|X| \equiv |X_G|$ (mod *p*).
7. Obtain the class equation of S_3.
8. Show that no group of order p^r for $r > 1$ is simple, where *p is* a prime.
9. How many different **homomorphisms** are there of a tree group of rank 2 onto Z_6 ?
10. Show that $(x, y : y^2 x = y, yx^{-} y = x)$ is a presentation of the trivial group of one element.
11. Consider the evaluation **homomorphism** $\phi_5 : Q[x]$ -4 **R. Find** six elements in the Kernel of ϕ_5.
12. Find all generation of the cyclic multiplicative group of units of the field Z_7.
13. Show that the fields R and C are not isomorphic.
14. Give an example to show that a factor ring of an integral domain may not be a field.

(14 x 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions.*
Each question carries 2 weightage.

15. Show that the finite indecomposable abelian groups are exactly the cyclic groups with order a power of a prime.

16. Let H be a normal subgroup of a group G, and let m = (G **H).** Show that *am* E H for every a ε G.

17. Find the center of S_3 x Z_5.

18. Give isomorphic refinements of the two series :

    {0} < **(18)** < **(3)** < Z_{72} and {0} < (24) < (12) < Z_{72}.

19. Let X be a G-set. Show that G_x is a subgroup of G for each *x* E X.

20. Find the number of distinguishable ways the edges of a square of cardboard can be painted if six colours of paint are available, assuming no colour is used more than once and the same color can be used on any number of edges.

21. Show that there are no simple groups of order $p^r m$, where *p is* a prime and *m* < *p.*

22. Write all polynomials of degree < 2 in Z_2 [x].

23. Show that a non-zero polynomial f *(x)* E F*[x]* of degree *n* can have at most *n* zeros in a field F.

24. Let R be a commutative ring and let a E R. Show that I_a = {x E R : *ax* = **0}** is an ideal of R.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 weightage.

25. Let H be a subgroup of a group G. Show that left coset multiplication is well defined by the equation (a H) *(b* H) = (a *b)* H iff left and right cosets coincide.

26. Show that if N is a normal subgroup of a group G and if H is any subgroup of G, then H v N = HN = NH and $\frac{HN}{N} \sim H/(H \cap N)$.

27, Let P_1 and P_2 be sylow p-subgroups of a finite group G. Show that P_1 and P_2 are conjugate. Verify this theorem for S_4 with *p* = 3.

28. Determine all groups of order 8 upto isomorphism.

(2 x 4 = 8 weightage)