D 72887 (Pages : 3) Name..................................

Reg. **No.............................**

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MT 1C 03—REAL ANALYSIS—I

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question has 1 weightage.

1. Construct a compact set of real numbers whose limit points form a countable set.
2. Define perfect set. Give an example of a perfect set which is not bounded.
3. Prove that the set of all interior points of a set E is open.
4. Prove that a uniformly continuous function of a uniformly continuous functions is uniformly continuous.
5. Is inverse of a bijective continuous function continuous ? Justify your answer.
6. Identify the type of discontinuity of the following function :

$$f(x) = \begin{cases} \sin \frac{1}{x} & (x \quad 0) \\ 0 & (x = 0) \end{cases}$$

at $x = 0$.

7. State Taylors theorem.
8. Evaluate $\lim\limits_{x} \frac{\sin x}{x}$.
9. Is mean value theorem real valued functions valid for vector valued functions ? Justify your answer.
10. Let f be a bounded real valued function defined on $[a, b]$ and $|f|$ be Riemann integrable on $[a, b]$. Is f Riemann integrable ? Justify your answer.
11. Let f be a bounded function and a be a monotonic increasing function on $[a, b]$. If the partition P' is a refinement of the partition P of [a, b], then prove that $U(P', f, a) \quad U(P, f, a)$.

Turn over

12. Let y be defined on $[0, 2\pi]$ by $\gamma(t) = e^{it}$. Prove that y is rectifiable.

13. Define uniform convergence.

14. Prove that every function in an equicontinuous family of functions is continuous.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *from the following* **ten** *questions.*
Each question has weightage 2.

15. Prove that finite intersection of open sets is open. Is it true in the case of arbitrary intersection ? Justify your answer.

16. Prove that infinite subset of a countable set is countable.

17. For x, y E R^1, let $d(x, y) = \max\{|x|\ |y|\}$. Prove that d is a metric. Which subsets of the resulting metric space are open ?

18. Let f be a continuous mapping of a metric space X into a metric space Y and let E be a dense subset of X. Prove that f(E) is a dense subset of $f(X)$.

19. Let f be a real valued uniformly continuous function on the bounded set E in $\mathbb{R}$. Prove that f is bounded on E.

20. Let f be a real valued differential function on (a, b). If $f'(x) = 0$ for all x E (a, b), then prove that f is a constant.

21. Let f be a bounded function and a be a monotonic increasing function on [a, $b]$. Prove that if f is Riemann-Steiltjes integrable with respect to a on $[a, b]$, then $|f|$ is Riemann-Steiltjes integrable with respect to a on $[a, b]$ and

$$\left| \int_a^b d\, d\alpha \right| - \qquad d\alpha.$$

22. Let f be Riemann integrable on [a, b] and let F be a differentiable function on [a, $b]$ such that F' = f. Prove that $\int_a^b f(x)\, dx$ = F (b) – **F** (a).

23. Let $\{f_n\}$ be a sequence of functions defined on E such that $|f_n(x)| < M_n$ for all n = 1, 2, . . . and $x \in E$. Prove that *In* converges uniformly on E if $\sum M_n$ converges.

24. For n = 1, 2 . . . and x real let $f_n(x) = \frac{x}{1 + nx^2}$. Show that $\{f_n\}$ converges uniformly.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *from the following* **four** *questions.*
Each question has weightage 4.

25. (a) Prove that a finite set has no limit points.

(b) Let P be a non-empty perfect set in $\mathbb{R}^k$. Prove that P is uncountable.

26. (a) Prove that compact subsets of a metric space are closed.

(b) Let E be a subset of the real line $\mathbb{R}^1$. Prove that E is connected if and only if it satisfies the following property : If $x \in E$, $y \in E$ and $x < z < y$, then $z \in E$.

27. (a) Let f be defined on $[a, b]$. If f has a local maximum at a point x and if $f'(x)$ exists, then prove that $f'(x) = 0$.

(b) Let f be a continuous function and α be monotonic increasing function on $[a, b]$. Prove that f is Riemann-Steiltjes integrable with respect to α on $[a, b]$.

28. If $\{f_n\}$ be a sequence of functions on E and if $f_n \to f$ uniformly on E, then prove that f is continuous on E.

(2 x 4 = 8 weightage)