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Reg. No..................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CUCSS)

Mathematics

MAT 1C 03—REAL ANALYSIS—I

(2010 Admissions)

Time : Three Hours Maximum . 36 Weightage

Part A (Short Answer Questions) (1 —14)

Answer **all** *questions.*
Each question has 1 weightage.

1. Construct a bounded set of real numbers with exactly one limit point.

2. For x, y ***ER***[1] let $d(x, y) = 1x^2 - y^2 1$. Is d a metric on R ? Justify your answer.

3. Let E be a non-empty set of real numbers which is bounded above and let y = sup E. Prove that

   y E E.

4. Is a finite set closed ? Justify your answer.

5. Prove that the limit of a function is unique.

6. Construct a function which has a simple discontinuity at every rational point.

7. Let f be a differentiable function on [a, b]. Prove that f is continuous on [a. b].

8. Let f be a continuous function and f 0 on $[a, b]$. If $J f\ dx = 0$, then prove that $f(x) = 0$ for all
   a

   x E $[a, b]$.

9. **Let f_p f_2 be bounded** functions and a be a monotonic increasing function on [a, b]. Prove that if f_2 are Reimann-Steiltjes integrable with respect to a on [a, b], then $f_1 + l_2$ is Reimann-Steiltjes integrable with respect to a on $[a, b]$.

10. Let f be a bounded function and a be a monotonic increasing function on $[a, b]$. If the partition P' is a refinement of the partition P of [a, b], then prove that.

$$L(I, f, \quad L(P', f, a).$$

11. Let γ be defined on [0, 2π] by $7(t) = e^{t}$. Prove that y is rectifiable.

Turn over

12. Give an example of a convergent series of continuous functions with a discontinuous limit.

13. Prove that uniformly convergent sequence of bonded functions is uniformly bounded.

14. Define equicontinuous family of functions and give an example of it.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *from the following ten questions* (15 —24).
Each question has weightage 2.

15. Prove that the set of all integers is countable.

16. Prove that compact subsets of a metric space are closed.

17. Let f be a continuous real valued function on a metric space X. Prove that the set $Z(f) = \{x \in X \; f(x) = 0\}$ is a closed subset of X.

18. Let [x] denote the largest integer less than or equal to x. What type of discontinuities does the function $[x]$ have ?

19. If f is a real valued differentiable function on (a, b). If $f'(x)$ 0 for all $x \in (a, b)$, then prove that f is monotonic increasing on (a, b).

20. Let f be a bounded function and a be a monotonic increasing function on $[a, b]$. Prove that if f is Reimann-Steiltjes integrable with respect to a on [a, b], then $|f$ is Reimann-Steiltjes integrable with respect to a on [a, b] and $\left| \int_a f \, d\alpha \right| \int_a |f| \, d\alpha.$

21. For $1 < s < \infty$, define $\zeta(s) = \sum_{n=1} \frac{}{}$. Prove that $(s) = s \int \frac{x - [x]}{x^{s+1}} dx$ where $[x]$ denote the greatest integer less than or equal to x.

22. For what values of x does the series $\frac{1}{1 + n^2 x}$ converge absolutely.

23. Prove that the series $\sum_{n=1} (1-1 \frac{x^n}{n^2}$ converges uniformly in every bounded interval.

24. Let K be compact, $f_n \in C(K)$ n = 1, 2, 3, ... and let $\{f_n\}$ be pointwise bounded and equicontinuous on K. Prove that $\{f_n\}$ is uniformly bounded on K.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *from the following four questions* (25 —28).
Each question has weightage 4.

25. (a) Prove that countable union of countable sets is countable.

(b) Prove that the cantor set is perfect.

26. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if f^{-1}(V) is open in X for every open set V in Y.

(b) Prove that continuous image of a connected space is connected.

27. (a) State Taylor's theorem.

(b) Let f be a bounded function, a be monotonic increasing function and a' is Reimann integrable on *[a, b]*. Prove that f is Reimann-Steiltjes integrable with respect to a on [a, *b]* if and only if f a' is Reimann integrable on *[a, b].*

28. Let γ be a curve on [a, b] and let γ' be continuous on [a, b]. Prove that γ is rectifiable and

$$\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt.$$

(2 x 4 = 8 weightage)