D 52974 (Pages : 4) Name...

Reg. No.................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014

(CUCSS)

Mathematics

MT IC 04—ODE AND CALCULUS OF VARIATIONS

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Determine the radius of convergence of the series $\sum_{n=1} \frac{p(p-1)(p-2)\ldots(p-n+1)}{n!} x^n$.

2. Locate and classify the singular points on the x-axis of the differential equation

   $x^3 (x-1) y'' - 2 (x-1) y' + 3xy = 0.$

3. The equation $y'' + \left(p + \frac{1}{2} - \frac{x^2}{4}\right) y = 0$, where p is a constant has a series solution of the form

   $y = \sum a_n x^n$. Show that the coefficients a_n are related by the formula

   $(n+1)(n+2) a_{n+2} + \left(p + \frac{1}{2}\right) a_n - \frac{1}{4} a_{n-2} = 0.$

4. Determine the nature of the point $x = \infty$ for Legendre's equation $(1 - x^2) y'' - 2xy' + p(p+1) y = 0$.

5. Verify that $P_n(-1) = (-1)^n$ where $P_n(x)$ is the nth degree Legendre polynomial.

6. Write Bessel's equation of order two and show that $x = 0$ is a regular singular point of it.

7. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

8. Describe the phase portrait of the system $\frac{dx}{dt} = x, \ \frac{dy}{dt} = 0$.

Turn over

9. **Determine whether the function $-x^2 - 4xy - 5y^2$ is positive definite, negative definite, or neither.**

10. **Show that every non-trivial solution of $y'' + (\sin x + 1) y = 0$ has an infinite number of positive zeros.**

11. **State Picard's theorem.**

12. **Show that $f(x, y) = xy$ does not satisfy a Lipschitz condition on any strip a x b and $-a < y < a$.**

13. **Find the extremal for** r $\frac{(1)^2}{}$ dx

14. **What is the isoperimetric problem.**

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Express sin x in the form of a power series $\sum a_n x^n$ **by solving $y' = (1 - x^2)$** $y(0) = 0$; in two ways.

16. Find the general solution $y'' + xy' + y = 0$.

17. **Show that the solutions of the equation $(1 - x^2) y''$** $2xy' + n(n + 1) y = 0$, where n is a non-negative integer, bounded near $x = 1$ are precisely constant multiples of F $\left[-n, n+1, 1, \frac{1}{2}(1 - x)\right]$.

18. Find the first three terms of the Legendre series of

$$f(x) = \begin{cases} 0 & if\ -1 \le x < 0 \\ x & if\ 0 \le x \end{cases}$$

Show that between any two positive zeros of $J_0(x)$ there is a zero of $J_1(x)$ and that between any two positive zeros of $J_1(x)$ there is a zero of $J_0(x)$.

19. **Find the critical points and the differential equation of the paths of the non-linear system :**

$$\frac{dx}{dt} \quad {}^{2} + 1), \quad \frac{dy}{dt} = 2xv^2$$

20. **If** $f(x) = \begin{cases} 1 & , 0 \le x < ½ \\ {}^1/_2 & , x = {}^1/_2 \\ 0 & , ½ < x \le 1, \end{cases}$

then show that $f(x) = \sum_{n=1}^{\infty} \frac{J_1 \left(\frac{\lambda_n}{2} \right)}{\lambda_n \, J_1(\lambda_n)^2} J_0(\lambda_n x)$, **where the** $\lambda_n' s$ **are the positive zeros of** $J_0(x)$.

21. **Verify that (0, 0) is a simple critical point of the system**

$\frac{dx}{dy} = x + y - 2xy, \ \frac{dy}{dt} = -2x + y + 3y^2$ **and determine its nature and stability properties.**

22. **State and prove sturm separation theorem.**

23. **Find the exact solution of the initial value problem** $y' = y^2$, **y (0) =1 : starting with** $y_0(x) = 1$, **apply Picard's method to calculate** $y_1(x)$, $y_2(x)$, $y_3(x)$, **and compare these results with the exact solution.**

24. **Find the curve of fixed length L that joins the points (0, 0) and (1, 0), lies above the x-axis and encloses the maximum area between itself and the x-axis.**

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*

Each question carries 4 *weightage.*

25. **Find two independent Frobenius series solutions of the equation** $x^2 y'' - x^2 y' + (x^2 - 2) y = 0$.

26. **Determine the general solution of the hyper geometric equation**

$$x(1 - x) y'' + [c - (a + b + 1) x] y' - aby = 0$$

Turn over

27. Find the general solution of the system $\frac{dx}{dt} = 7x + 6y$, $\frac{dy}{dt} = 2x + 6y$.

28. Let $f(x, y)$ be a continuous function that satisfies a Lipschitz condition

$$|f(x, y_1) - f(x, y_2)| \le k|y_1 - y_2|$$

on a strip defined by $a \le x \le b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $a < x < b$.

(2 x 4 = 8 weightage)