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Name....................................

Reg. No.................................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 04—ODE AND CALCULUS OF VARIATIONS

(2010 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer all *questions.*
Each question carries 1 weightage.

1. Define radius of convergence of a power series and determine the same for the power series :

$$\sum_{n=1} \frac{p(p-1) \cdots (p-n+1)}{n!} x^n ; p \neq 0.$$

2. Locate and classify the singular points on the x-axis for the equation $(3x+1)xy'' - (x+1)y' + 2y = 0$.

3. Evaluate $\lim_{b \to \infty} F\left(a, b, a; \frac{x}{b}\right)$.

4. Determine the nature of the point $x = \infty$ for Legendre's equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$, where p is a constant.

5. Find the first two terms of the Legendre series of $f(x) = e^x$.

6. Define gamma function and show that $\Gamma(n+1) = n!$ for any integer $n > 0$.

7. Show that $\frac{d}{dx}\left[x J_1(x)\right] = x J_0(x)$.

8. Describe the phase portrait of the system $\frac{dx}{dt} = 1, \frac{dy}{dt} = 2$.

Turn over

9. State Liapunov's theorem.

10. Show that a function of the form $a\,x^3 + b\,x^2 y + c\,xy^2 + d\,y^3$ cannot be either positive definite or negative definite.

11. Find the normal form of Bessel's equation $x^2 y'' + xy' + (x^2 - p^2)$

12. State Picard's theorem.

13. Show that $f(x, y) = y^{1/2}$ satisfies a Lipschitz condition on the rectangle $|x| \leq 1$ and $\frac{1}{2} \leq y \leq 1$.

14. Find the stationary function of $\int_0^4 \left[xy' - (y')^2\right] dx$ which is determined by the boundary conditions $y(0) = 0$ and $y(4) = 3$.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Show that $\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$ by solving $y' = 1 + y^2$, $y(0) = 0$ in two ways.

16. Find the general solution of :

$$(1 - e^x)\, y'' + \frac{1}{2} y' + e^x y = 0$$

near the singular point $x = 0$.

17. Show that the solutions of the equation $(1 - x^2)\, y'' - 2xy' + n(n+1)\, y = 0$, where n is a non-negative integer, bounded near $x = 1$ are precisely constant multiples of the polynomial :

$$F\left[-n, n+1, 1, \frac{1}{2}(1-x)\right].$$

18. If $1(x) = x^p$ for the interval 0 x < 1, show that its Bessel series in the functions $J_p(\lambda_n x)$, where the X, 's are the positive zeros of J_p (x), is $" = \sum_{n=1} \frac{}{\lambda_n J_{p+1}(\lambda_n)} J_p(\lambda_n x)$.

19. Show that if w *(t)* is the Wronskian of the two solutions $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ on *[a, b]* of the homogeneous system $\frac{dx}{dt} = (t)\,x + b_1(t)\,y\,;\ \frac{dy}{dt} = a_2(t)\,x + b_2(t)\,y$, then w *(t)* is either identically zero or nowhere zero on *[a, b].*

20. Determine the nature and stability properties of the critical point **(0, 0)** for the system :

$\frac{}{dt} = -4x - y,\ \frac{dy}{dt} = x - 2y.$

21. Show that (0, 0) is an unstable critical point for the system $\frac{dx}{dt} = 2xy + x^3\ \ \frac{dy}{dt} = -x^2 + y^5$

22. If y (x) is a non-trivial solution of $y'' + q(x)\,y = 0$, show that y (x) has an infinite number of positive zeros if $q(x)\ \frac{k}{x^2}$ for some $k > \frac{1}{4}$, and only a finite number if $q(x) < \frac{1}{4x^2}$.

23. A curve in the first quadrant joins (0, 0) and (1, 0) and has a given area beneath it. Show that the shortest such curve is an arc of a circle.

24. Explain Picard's method of successive approximations of solving the initial value problem :

$y = f(x, y),\ y(x_0) = y_o,$ where $f(x, y)$ is an arbitrary function defined and continuous in some neighbourhoood of the point (x_0, y_0)

(7 x 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries 4 *weightage.*

25. Find two independent Frobenins series solutions of the equation :

$x^2 y'' + xy' + (x^2 - \frac{1}{4})\,y = 0.$

Turn over

26. State and prove the orthogonality property for $J_p(x)$, the Bessel functions of order p.

27. For the non-linear system :

$$\frac{dx}{dt} \quad y(\;^{2} + 1) \quad \frac{dy}{dt} = 2xy \,.$$

(i) Find the critical points.

(ii) Find the differential equation of the paths.

(iii) Solve this equation to find the paths.

(iv) Sketch a few of the paths.

28. Solve the initial value problem by Picard's method :

$$\frac{dy}{dx} = z \qquad y(0) = 1$$

$$\frac{dz}{dx} = -y \qquad z(0) = 0.$$

(2 x 4 = 8 weightage)