D 52972 (Pages : 3) Name........................................

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# FIRST SEMESTER M.Sc. DEGREE EXAMINATION, JANUARY 2014

(CUCSS)

Mathematics

## MT IC 02—LINEAR ALGEBRA

Time : Three Hours Maximum : 36 Weightage

### Part A (Short Answer Type)

*Answer* **all** *questions.*
*Each question has weightage 1.*

1. Prove that for a, $_3$ in a vector space V, -(a +13) = -a + (-β).

2. Show that U = {(x, 2x) : $x$ e RI is a subspace of $\mathbb{R}$ .

3. Verify whether the set of all 3 x 3 diagonal matrices span the space of all 3 x 3 matrices.

4. Find the dimension of the space of all upper triangular 2 x 2 matrices over R .

5. Find the Co-ordinate vector of (1, -1, 2) E $R^3$ w.r.t. the ordered basis {(1, 0, 1), (1, 0, 0), (1, 1, 0)}.

6. Let T : $R^2 \to R^3$ be defined by T(x, y) = $(x + y, xy, y)$ . Verify whether T is linear.

7. Let W = span {(1, 0, 1), (0, 0, 1)}. Let $f: R^3 \to R$ be defined by :

$f(x, y, z) = y$ . Verify whether $f$ E $W^\circ$ •

8. Find the characteristic polynomial of [1 2 / 2 3 .

9. Find all characteristic values of $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

10. Let W = Span ((1, 2, 1)} and T : $R^3$ $R^3$ be defined by T(x, y, $z$) = $(x + z, 2y,$ 2z). Verify whether W is an invariant subspace of T.

11. Let $W_1$ = span {(1, 1, 1), (1, 1, 2)), and $W_2$ = span {(2, 2, 3)). Verify whether $W_1 + W_2$ is a direct sum.

12. Verify whether T $R^2$ _ defined by T(x, y) = x ± y x z y is a projection.

Turn over

13. Verify whether **(1, 1), (1,** -1) are orthogonal in $R^2$.

14. Prove that if E is an orthogonal projection of V onto W then $Im(1 - E) = W^{\perp}$.

(14 x **1** = 14 weightage)

**Part B (Paragraph Type)**

*Answer any* **seven** *questions.*
*Each question carries weightage* 2.

15. Let V be a vector space over a field F and c 0. Prove that if ca = 0 for some a E V, then a = 0.

16. Verify whether $S = \{(x, y, x + y + 1) : x, y \in \mathbb{R}\}$ is a subspace of

17. Show that the row space of $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{vmatrix}$ is **R3.**

18. Show by an example that for non-zero subspaces $\mathbf{W_1}, \mathbf{W_2}$ of a vector space V, dim $(W_1 + W_2)$ can be equal to dim $W_1$.

19. Find the matrix of the transformation T $R^3 \to R^3$ defined by $T(x, y, z) = (x + y, y + z, y)$ w.r.t. the ordered basis ((0, 1, 1), (1, 1, 0), (1, 0, 0)1.

20. Let $V = R^2$ and B = {(1, 0), (0, 1)}. Find the dual basis of $V^*$ corresponding to B.

21. Let T be an invertible linear operator and $c \neq 0$ be a characteristic value: of T. Prove that $\frac{-}{c}$ is a characteristic value of $T^{-1}$.

22. Express **le** as a direct sum $W_1$ $\mathbf{W_2}$, where $W_1$ = span ((1, 1, 1), (1, 2, 1)).

23. Let T be a linear operator on V and let $V = W_1 \oplus \mathbf{W_2}$, where $W_1$ and $\mathbf{W_2}$ are invariant under T. Prove that $\det(T) = \det(T_1) \cdot \det(T_2)$ where $T_i$ is the restriction of T to $W_i$ for $i = 1, 2$.

24. Verify whether (x y) defined by $(x \mid y) = x1 + \bar{y_1}$ for $x = (x_1, x_2)$, $y = (y_1, y_2)$ is an inner product on $R^2$.

(7 x 2 = 14 weightage)

## Part C (Essay Type)

*Answer any* **two** *questions.*
*Each question carries weightage* 4.

25. Define Co-ordinate vector of an element a in a finite dimensional vector space V, over a field F. Let B be an ordered basis of V and P be an n x n invertible matrix over F where $n = \dim V$. Prove that there exists a unique ordered basis B' of V such that $[\alpha]_B = P[\alpha]_{B'}$ for all a E V

26. (a) Let {a$_1$, $\alpha_2,\ldots,\alpha_n$ I be a basis of a vector space V and $\beta_1$, P$_2$ ,$\beta_n$ be elements of V. Prove that there exists a unique linear operator T on V such that $T(\alpha_i) = \beta_i$ for every $i$.

    (b) Find all linear operators on the one-dimensional space III over R .

27. Define the transpose $T^t$ of a linear transformation T. Prove that for linear transformations of finite dimensional spaces, rank (T) = rank (Ti).

28. Let V be a finite dimensional space and $W_1$, $W_2,\ldots,W_k$ be subspace of V and let $V = W_1 + \ldots + W_k$. Prove that the following are equivalent

    (a) $W_1$, $W_2,\ldots,W_k$ are independent.

    (b) If $B_i$ is a basis of $W_i$ for $i$ = 1,.., $k$ then $B = B_1 \cup$ B2 v ... v $B_k$ is a basis for V.

(2 x 4 = 8 weightage)