

D 92951

(Pages : 2)

Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

Time : Three Hours

Maximum : 36 Weightage

Part A (Short Answer Type)

*Answer all the questions.
Each question carries weightage 1.*

1. Write an example for one dimensional vector space.
2. Show that the null space of a linear transformation is a subspace of the domain space.
3. Define basis for a vector space.
4. For any two real numbers x and y , let $S = \{(x, y) : 2x + 3y + 4 = 0\}$. Is the set S a subspace of the two dimensional Euclidean space ? Justify your claim.
5. Define $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by the rule $T(x, y) = (1 + x, y)$. Is T a linear transformation ? Justify your claim.
6. Define linear functional and give an example for a linear functional.
7. Define hyperspace of a vector space.
8. If S is a subset of a finite dimensional vector space V , with usual notation prove that $(S^0)^0$ is the subspace spanned by S .
9. If A and B are two matrices that are similar, what can we say about their characteristic polynomials ? Justify your claim.
10. Prove that \mathbb{R} is a subspace of the inner product space \mathbb{R}^3 with usual inner product.
11. Write the relation between linear transformation and matrices in finite dimensional vector spaces.
12. Define projection map. Show that projection map is diagonalizable.
13. When do we say that a matrix A is the direct sum of the n matrices $A_1, A_2,$
14. Show that an orthogonal set of non-zero vectors is linearly independent.

(14 x 1 = 14 weightage)

Turn over

Part B (Paragraph Type)

Answer any **seven** questions.
Each question carries **weightage 2**.

15. Prove that in a finite dimensional vector space, every non-empty linearly independent set of vectors is a part of a basis.
16. Let V and W be vector spaces over the field F and T be a linear transformation from V to W . If T is invertible, then show that the inverse function T^{-1} is a linear transformation from W onto V .
17. Prove that every n -dimensional vector space over the field F is isomorphic to the space
18. Let V and W be vector spaces over the field F and T be a linear transformation from V into W . Suppose that V is finite-dimensional. Then prove that $\text{rank}(T) + \text{nullity}(T) = \dim V$.
19. Let $B = \{a_1, a_2, a_3\}$ be the ordered basis for \mathbb{R}^3 consisting of $a_1 = (1, 0, -1)$, $a_2 = (1, 1, 1)$, $a_3 = (1, 0, 0)$. What are the co-ordinates of a vector (a, b, c) in the ordered basis B ?
20. Let V be a finite dimensional vector space over the field F . Then prove that each basis of V^* is the dual of some basis of V .
21. Let V be a vector spaces over the field F . If L is a linear functional on the dual space V^* of V , prove that there is a unique vector a in V such that $L(f) = f(a)$.
22. Let V and W be finite dimensional vector spaces over the field F . Prove that V and W are isomorphic if and only if $\dim V = \dim W$.
23. State and prove the Cauchy-Schwarz inequality.
24. Prove that every finite dimensional inner product space has an **orthonormal** basis.

(7 x 2 = 14 weightage)

Part C (Essay Type)

Answer any **two** questions.
Each question carries **weightage 4**.

25. Let V be an m -dimensional vector space over the field F and W be an n -dimensional vector spaces over F . Then with usual assumptions prove that the space $\mathbf{L}(V, W)$ is a finite-dimensional vector space of dimension mn .
26. State and prove the **Cayley** - Hamilton theorem for a linear operator on a finite dimensional vector space.
27. Define characteristic polynomial $f(x)$ and minimal polynomial $p(x)$ of a linear operator on an n -dimensional vector space V . Show that $f(x)$ and $p(x)$ have the same roots except for multiplicities.
28. Check whether the operator T given by $T(x, y, z) = (x + y + z, 2y + z, 3z)$ is **diagonalizable** or not.

(2 x 4 = 8 weightage)