

D 13183

(Pages : 3)

Name.....

Reg. No.....

FIRST SEMESTER M.Sc. DEGREE. EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 02—LINEAR ALGEBRA

(2010 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A (Short Answer Type)

Answer **all** the questions.
Each question carries **weightage 1**.

1. Write an example for a non-trivial subspace of the two dimensional **Euclidean** space ? Justify your claim.
2. Define $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by the rule $T(x, y) = (\sin x, y)$. Is T a linear transformation ? Justify your claim.
3. Define basis of a vector space.
4. Let V be a finite dimensional vector space. What is the minimal polynomial for the identity operator on V.
5. Describe explicitly an inner product on \mathbb{R} , the set of real numbers.
6. Let F be a field and let T be the linear operator on F^2 defined by $T(x_1, x_2) = (x_1 + x_2, x_1)$. Prove that T is non-singular.
7. Define **hyperspace** of a vector space.
8. Define inner product space.
9. Prove that \mathbb{R}^2 is a subspace of the inner product space \mathbb{R}^3 with usual inner product.
10. Verify whether the vectors $(1, 2), (-2, 1)$ are orthogonal in \mathbb{R}^2 .
11. Find the characteristic polynomial of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.
12. Suppose that $T\alpha = c\alpha$. If f is any polynomial, prove that $f(T)\alpha = f(c)\alpha$.
13. State the **Cayley-Hamilton** theorem.
14. Show that an orthogonal set of non-zero vectors is linearly independent.

(14 x 1 = 14 weightage)

Turn over

Part B (Paragraph Type)

*Answer any seven questions.
Each question carries **weightage** 2.*

15. Prove that the intersection of any two sub spaces of a vector space V is again a sub space of V .
16. Let $[a, b]$ be a closed interval on the real line and $C([a, b])$ be the space of all continuous real valued functions on $[a, b]$. Then prove that $L(g) = \int_a^b g(t) dt$ defines a linear functional on $C([a, b])$.
17. If W_1 and W_2 are finite dimensional subspaces of a vector space V , then prove that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$.
18. If A is an $m \times n$ matrix with entries in the field F , then prove that $\text{row rank}(A) = \text{column rank}(A)$.
19. Let T be the linear operator on \mathbb{R}^2 defined by $T(x_1, x_2) = (x_2, x_1)$. What is the matrix of T in the standard basis for \mathbb{R}^2 ?
20. Define annihilator of a subset of a vector space. State and prove the relation that connects the dimensions of a vector space, that of its subspace and of the annihilator space of this subspace.
21. Define transpose of a linear transformation from one vector space to another. If the vector spaces are finite dimensional, prove that rank of a linear transformation is equal to the rank of its transpose.
22. If A and B are $n \times n$ complex matrices, show that $AB - BA = I$ is impossible.
23. Define characteristic polynomial of a matrix. Prove that similar matrices have the same characteristic polynomial.
24. Prove that if E is a projection of \mathbb{R}^n along N , then $(I - E)$ is a projection of \mathbb{R}^n along N .

(7 x 2 = 14 **weightage**)

Part C (Essay Type)

*Answer any two questions.
Each question carries **weightage** 4.*

25. Let V be an m -dimensional vector space over the field F and W be an n -dimensional vector spaces over F . Then with usual assumptions prove that the space $L(V, W)$ is a finite-dimensional vector space of dimension mn .

26. Prove that every finite dimensional inner product space has an **orthonormal** basis.
27. Let W be a finite dimensional subspace of an inner product space V and E be an orthogonal projection of V on W . Then prove that E is an **idempotent** linear transformation of V onto W , W^\perp is the null space of E , and $V = W \oplus W^\perp$.
28. Let W be a finite dimensional subspace of an inner product space V and E be an orthogonal projection of V on W . Then prove that $I - E$ is the orthogonal projection of V on W^\perp . Also prove that $I - E$ is an **idempotent** linear transformation of V onto W^\perp with null space W .

(2 x 4 = 8 weightage)