D 92954 **(Pages : 3)** **Name**

Reg. No.....................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2015

(CUCSS)

Mathematics

MT 1C 05—DISCRETE MATHEMATICS

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 weightage.

1. Let X be a set and $<$ be a binary relation on X which is reflexive and transitive. Define a binary relation R on X by x R y if and only if x y and y x. Is R an equivalence relation ? Justify your claim.

2 Define a Boolean algebra. Give an example of a finite Boolean algebra.

3. Define atoms of a power set Boolean algebra P (X). Illustrate it with an example.

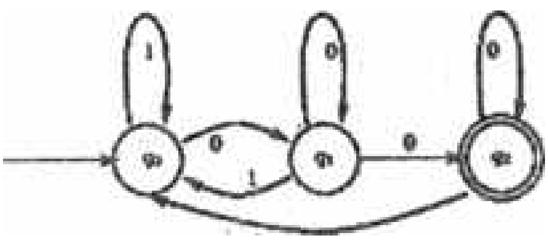
4. Write the conjunctive normal form of $(xy + x'y + x' \;\; (x + y)$.

5. Define a dfa and its transition graph. Illustrate.

6. Find a dfa for the language $L = \{a^n b : n > 0\}$.

7. Find a nfa which accepts the set of all strings containing '*aabb*' as a substring.

8. Find the language accepted by the following automaton :



9. Define Petersen graph. Find the girth of Petersen graph.

10. Prove that every u v walk contains a u v path.

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11. In any graph G, prove that $\delta(G) \leq 2 \frac{e(G)}{n(G)} \leq \Delta(G)$.

12. In any graph G with $n(G) > 1$, prove that every minimal disconnecting set of edges is an edge cut.

13. If G is a plane graph then prove that $\sum l(F_i) = 2e(G)$, where $l(F_i)$ denotes the length of the face F.

14. Find a graph G in which the strict inequality $k(G) < k'(G) < \delta(G)$, holds.

(14 x 1 = 14 weightage)

Part B

Answer any **seven** *from the following ten questions (15-24).*
Each question carries 2 weightage.

15. If x and y are elements of a Boolean algebra, prove that $x = y \Leftrightarrow xy' + x'y = 0$.

16. Let $(X_1, \leq_1)$ and $(X_2, \leq_2)$ be partially ordered sets. Define $\leq$ on $X_1 \times X_2$ by $(x_1, x_2) \leq (y_1, y_2)$ if and only if $x_1 \leq_1 y_1$ and $x_2 \leq_2 y_2$. Verify whether $(X_1 \times X_2, \leq)$ is a partially ordered set. Is it totally ordered ? Justify your claim.

17. Prove that the power set of any set partially ordered by inclusion is a lattice.

18. Find a regular expression for the language L $\{w \in \{0,1\}^* :$ w has no pair of consecutive zeros$\}$.

19. Find a regular expression for the language $L = \{a^n b^m : n + m$ is even$\}$.

20. Prove that every closed walk contains an odd cycle.

21. Characterize a family of graphs G_i with $k(G_i) = k'(G_i)$.

22. Define self complementary graph and decomposition of a graph. Describe a relation between them and illustrate it with an example.

23. Determine the values of m and n such that $K_{m,n}$ is Eulerian.

24. Let G be a connected graph with at least three vertices. Form G' from G by adding an edge with endpoints x, y whenever $d_G(x, y) = 2$. Prove that G' is 2-connected.

(7 x 2 = 14 weightage)

Part C

Answer any **two** *from the following four questions (25-28).*
Each question carries **4** *weightage*

25. Let $(X, +, \cdot, ')$ be a Boolean algebra. Prove that

    (a) Every non-zero element of X contains at least one atom.

    (b) Every two distinct atoms of X are mutually disjoint.

26. Let L be the language accepted by the nfa $M_N = (Q_N, E, \delta_N, q_0, F_N)$. Prove that there exists a dfa $M_D = (Q_D, E, \delta_D, \{q_0\}, F_D)$ such that $L = L(M_D)$.

27. (a) Prove that every connected graph contains a spanning tree.

    (b) If every vertex of a graph G has degree at least three, prove that G has a cycle of even length.

28. (a) If G is a simple graph prove that $k(G) \leq k'(G) \leq \delta(G)$.

    (b) Determine all *r* and s such that $K_{r,s}$ is planar.

(2 x 4 = 8 weightage)