D 72889 (Pages : 3) Name..................................

Reg. No..............................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2014

(CUCSS)

Mathematics

MAT 1C 05—DISCRETE MATHEMATICS

Time : Three Hours —————— Maximum : 36 Weightage

Part A (Short Answer Questions) (1 – 14)

Answer all questions.
Each question carries 1 weightage.

1. Define strict partial order and give an example of it. If R is a partial order on a set X, then prove that R – {(x, x): x E X} is a strict partial order on X.
2. Prove that intersection of two chains is a chain.
3. Let (X, + ,.,') be a Boolean algebra. Prove that $x + x = x$ for all x E X.
4. Prepare the table of values of the following function :

   $f(x_1, x_2, x_3) = x_1 x_2 (x_1 + x_2 + x_1 x_3)$.
5. Define Chromatic number of a graph. Find the chromatic number of P_5.
6. Prove that every graph with n vertices and k edges has at least n – k components.
7. If every vertex of a graph G has degree at least 2, then prove that G contains cycle.
8. Prove that every tree with at least two vertices has at least two end leaves.
9. Define Connectivity of a graph. Prove that k (KO = n -1.
10. Is every subgraph of a non-planar graph non-planar ? Justify your answer.
11. Let u be a string on the alphabet E. Prove that $|u$ I = n $|u|$ for all n =1, 2, ——
12. Let G = ({S}, {a, b}, S, P) be a grammar with productions P given by

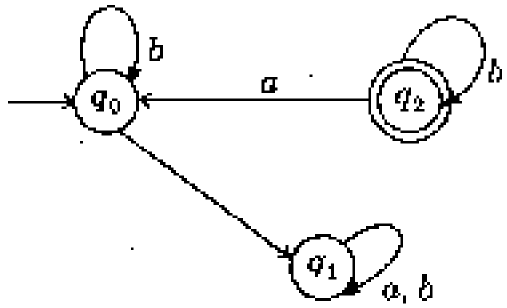
    S → aA, A → bS, S → X.

    Give a simple description of the language generated by G.

Turn over

13. Define non-deterministic acceptor and give an example of it.

14. Find the set of strings accepted by the following deterministic finite acceptor.



(14 x 1 = 14 weightage)

Part B

Answer **any seven** *from the following ten questions* **(15 – 24).**
Each question carries weightage **2.**

15. **Let (X, +, . ,') be a Boolean algebra. Prove that the corresponding lattice (X, is complemented and distributive.**

16. **Let (X, +, . ,') be a finite Boolean algebra. Prove that every non-zero element of X contains at least one atom.**

17. **Prove that the characteristic numbers of a symmetric Boolean function completely determine it.**

18. **Prove that Petersen graph has diameter 2.**

19. **Prove that every, u, v-walk contains a u, v-path.**

20. **Let G be a graph. Prove that**

$$\delta(G) < \frac{2e(G)}{n(G)} < \Delta(G),$$

here $e(G)$ and $n(G)$ denote the number of edges and vertices in G respectively.

21. **Draw a graph G with** $k(G) < k'(G) < \delta(G)$.

22. **Is Euler's formula valid for a disconnected graph ? Justify your answer.**

23. **Find a grammar that generate the language {a^{n+2} b: n**

24. **Construct a nondeterministic acceptor that accepts the language** $\{ab, abc\}^*$.

(7 x 2 = 14 weightage)

Part C

Answer any two from the following four questions. **(25 – 28)**
Each question carries weightage **4.**

25. (a) Let $(X, +, \cdot, ')$ be a finite Boolean algebra. Prove that every element of X can be uniquely expressed as sum of atoms.

(b) Write the Boolean function :

$$f(a, b, c) = a + b + c'.$$

in their disjunctive normal form.

26. (a) Prove that a graph is a bipartite graph if and only if it has no odd cycle.

(b) Let G be a graph. Prove that

$$\sum_{v \in V(G)} d(v) = 2e(G).$$

27. Let G be an n-vertex graph with n 1. Prove that the following are equivalent :

(a) G is connected and has no loops.

(b) G is connected and has n —1 edges.

(c) G has n —1 edges and no cycles.

(d) G has no loops and has, for each u, v $\in V(G)$, exactly one u, v-path.

28. Define equivalent grammars. Prove that the grammar G = {a, b}, S, P) with productions P given by :

$$S \rightarrow SS \mid SSS \mid aSb \mid bSa \mid \lambda,$$

is equivalet to the grammar G' = ({S}, {a, b}, S, P') with production p' given by :

$$S \rightarrow SS \mid aSb \mid bSa \mid \lambda.$$

(2 x 4 = 8 weightage)