D 13191 (Pages : 3) Name...

Reg. No

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, DECEMBER 2016

(CUCSS)

Mathematics

MT 1C 05—DISCRETE MATHEMATICS

(2016 Admissions)

Time : Three Hours Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer all questions.
Each question has weightage 1.

1. Compute $|E(G)| + |E(G^C)|$ for a graph on n vertices.
2. If a simple graph G is not connected, prove that GC is connected.
3. Define identity graph. Illustrate with an example.
4. Define connectivity and edge connectivity. Give an example.
5. If $e = xy$ is not a cut edge of the graph G, prove that e belongs to a cycle of G.
6. Show that 6 (G) 5, if G is a simple planar graph.
7. Give an example of a poset with no maximum element and with exactly one maximal element.
8. Prove *or* disprove. The union of two chains in a poset is a chain.
9. Define a strict partial order. If P is a partial order on the set X, show that P – $\{(x, x) : x \mathbf{E} \mathbf{X}\}$ is a strict partial order.
10. Define a Boolean function of n variables. Give an example of a Boolean function of 3 variables.
11. Let $E = \{a, b, c\}$ and $L = \{a, b\}$. Find L+ and L^2.
12. Design a *dfa* which accepts string 1100 only.
13. If E {0, 1}, design an *nfa* to accept set of strings ending with two consecutive zeros.
14. Find an *nfa* which accepts the set of all strings containing *aabb* as a substring.

(14 x 1 = 14 weightage)

Turn over

Part B

Answer any **seven** *questions from the following ten questions.*
Each question has weightage 2.

15. If every vertex of a graph G has atleast degree 2, prove that G contains a cycle.

16. Prove that every edge of a tree is a cut edge.

17. Prove that every connected graph contains a spanning tree.

18. If G is a plane graph and *f is* a face of G prove that there exists a plane embedding of G in which *f* is the exterior face.

19. Prove that $\mathbf{K_5}$ is non-planar.

20. Define total order. Give an example of a partial order which is not a total order.

21. Let (X, +, ·,′) be a Boolean algebra. Prove that $x + 1 = 1$ and $x \cdot 0 = 0$.

22. Prepare the table for values of the function $f(x_i, x2) = x_1 x_2 + x_1 x_2' +$

23. Design an *nfa* with three states that accepts the language $L = \{ab, abc\}^*$.

24. Find a *dfa* for the language L = a" : n is odd, n ≠ 3}.

(7 x 2 = 14 weightage)

Part C (Essay Type)

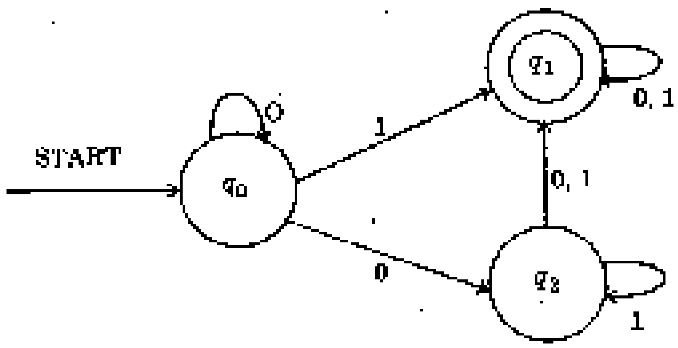
Answer any **two** *questions from the following four questions.*
Each question has weightage **4.**

25. For any loopless connected graph G, prove that x (G) X (G) s 8 (G).

26. Prove that a graph is planar if and only if each of its blocks is planar.

27. Let (X, +, ·,′) be a Boolean algebra. If x, y E X define $x \text{ s } y$ if $x \cdot y' = 0$. Prove that (X, is a lattice. Find the maximum and minimum elements of this lattice.

28. Convert the *nfa* given by the transition graph into an equivalent *dfa* :



(2 x 4 = 8 weightage)