D 33332 (Pages : 2) Name

Reg. No..............................·····

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, FEBRUARY 2013

(CUCSS)

Mathematics

MT 10 02—LINEAR ALGEBRA

(2010 admissions)

Time : Three Hours —————— Maximum : 36 Weightage

Part A (Short Answer Type)

Answer **all** *questions.*
Each question carries weightage 1.

1. Let V be a vector space over a field F. Prove that if 0 e V and C E F then 0.0 = 0.
2. Prove that $U = \{(x, x) : x \in \mathbb{R}\}$ is a subspace of R^2.
3. Verify whether the set of all upper triangular matrices span the space of all 2 x 2 matrices.
4. Find the dimension of the space of all n x n diagonal matrices over R ·
5. Find the co-ordinate vector of (1,2,3)e with respect to the ordered basis 41,2,0), (1,1,0), (0,1,1)}.
6. Verify whether $f(x, y) = xy$ for $(x, y) \in R^2$ is a linear transformation from R^2 to R.
7. Let W = span {(1,1, 0), (1, 0,1)}. Let f be defined by $I(x, y, z) = x - y - z$. Verify whether f belongs to W^0.
8. Find the characteristic polynomial of $\begin{bmatrix} & \\ 3 & 4 \end{bmatrix}$
9. Find the characteristic values of $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$
10. Let W = span $\{(1,1,1)\}$ in R^3 and $T : R^3 \to R^3$ be defined by $T(x, y, z) = (x + y, y + z, 2z)$. Verify whether W is an invariant subspace of T.
11. Let W_1 = span $\{(1,2,1)\}$ and W_2 = span $\{(2,1,1)\}$. Verify whether $W_1 + W_2$ is a direct sum.
12. Verify whether $T : R^2 \to R^2$ defined by T (x, y) = (2x + y, 0) is a projection.
13. Verify whether (1,2), (-2,1) are orthogonal in R^2.

if E is an orthogonal projection of a space V on a subspace W, prove that Null (1 – E) = W.

(14 x 1 = 14 weightage)

Turn over

Part B (Paragraph Type)

- *Answer any* **seven** *questions.*
Each question carries weightage 2.

15. Let S ; $\{\alpha, \beta\}$ be a subset of V, where V is a vector space. Prove that the set of all linear combinations of S is a subspace of V.

16. Verify whether S = {(x, y, xy) : x e R} is a subspace of $\mathbb{R}^3$.

17. **If** W_1, W_2 are subspaces of a vector space V, prove that $W_1 + W_2$ is a subspace of V.

18. Let V be a vector space of dimension n. Prove that any set of n — **1** vectors is not a basis of V.

19. Find the matrix of the linear transformation T $\mathbb{R}^{-}$ $\mathbf{R}^3$ defined by T (x, y, z) = $(x + y, x, y + z)$ with respect to the ordered basis B = 41,2,1),**(1,1, 2)**, $(2,1,1)\}$.

20. Let $\{\alpha_1, \alpha_2, \ldots, a_n\}$ be a basis of a vector space V and f_2 f_n be the dual basis of VA. Prove that $\alpha =$ f_i (a) α_i for each a e V •

21. Let T be a linear operator on a vector space V and for some a e V let T $(a) = ca$. Prove that for any polynomial f, f (T) (c) $.\alpha$

22. Express R^3 as a direct sum $W_1 \oplus W_2$, where W_1 = span $\{(1,1,1)\}$.

23. Let T be a linear operator on a vector space V = $W_1 \oplus \mathbf{W_2} \oplus \ldots$ $\mathbf{W_k}$; where each W_i is an invarian subspace for T. Prove that if each W_i is an eigen space of V then T is diagonalizable.

24. Verify whether (x $y)$ given by $(x$ $y) = x_1y_1$ x_2y_2 for x =), y = (y_1, y_2) **E** $\mathbf{R}^2$ is an inner product.

(7 x 2 = 14 weightage)

Part C (Essay Type)

Answer any **two** *questions.*
Each question carries weightage 4.

25. Define dimension of a vector space. Show that if W_1, W_2 are subspaces of a finite dimensional vector space, then dim$(W_1 + \mathbf{W_2})$ = dim W_1 + dim $\mathbf{W_2}$ – dim $(W_1\ n\ W_2)$.

26. Let A be an m x n matrix over a field F. Show that for any n x 1 matrix X, T (X) = AX is a linear transformation from the space of n **x 1** matrices to the space of m **X** 1 matrices. Prove also that rank T = column rank of A.

27. Define the annihilator W° of a subspace W. Let W_1, W_2 be subspaces of a finite dimensional space V. Prove that if W_1^- + W° then $W_1 = W_2$.

28. Define projection. Show that if E is a projection on a vector space V then V = R $\oplus$ N where R is range of E and N is the null space of E.

(2 **x** 4 = 8 weight