C 80270 (Pages : 4) Name.................................

Reg. No.................................

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS—UG)

Mathematics

MAT 6B 10—COMPLEX ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Define an analytic function and give an example of a function which is not analytic at the origin.
2. Fill in the blanks : The locus of the points z satisfying $|z+2i|^2 = 2|i+1|$ is a/an ————.
3. Verify whether $f(z) = \bar{z}/z$ is analytic or not at $z = 0$?
4. Find the simple poles, if any for the function $f(z) = \dfrac{(z-1)^2}{z^3(z^2+9)}$.
5. Is $u(x, y) = x^2 - y^2 + xy$ a harmonic function ? Justify your claim.
6. Define essential singularity of a complex valued function.
7. Fill in the blanks : The real part of $\log(2z)$ is ————.
8. Write the formula for the evaluation of n^{th} derivative of an analytic function with full assumptions involved.
9. Solve for z : $3z - 1 = 2\bar{z}$.
10. If R is the radius of convergence of $\sum a_n z^n$, find the radius of convergence of $\sum a_n z^{3n}$.
11. What do you mean by a Jordan curve ?
12. Find the value of $i^i + \text{Log}(2i)$.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 *marks.*

13. Evaluate the line integral of $f(z) = z^2$ over the line joining $2i$ to $i - 1$.

14. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.

15. Show that $\tan^{-1}(z) = \frac{i}{2} \log \frac{i + z}{i - z}$.

16. Show that the poles of an analytic function are isolated.

17. Which one is bigger : $\left\| z_1 \right| - \left| z_2 \right\|$ *or* $\left| z_1 - z_2 \right|$. Prove your claim.

18. Find the radius of convergence of the power series : $\sum_{n=0}^{\infty} \frac{n!\, z^n}{n^n}$.

19. Verify Cauchy-Groursat theorem for $f(z) = z^5$ when the contour of integration is the circle with centre at origin and radius 3 units.

20. Locate the poles and zeros, if any, of $f(z) = \sin(1/z)$ in the complex plane.

21. Find all the solutions of $e^z = 2$.

22. Find the residue of $f(z) = \sin(z)/z^2$ at $z = 0$ and evaluate the integral of $f(z)$ around the circle containing zero inside it.

23. Using the definition of continuity show that $\sin z$ is continuous through out the plane.

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Find the real and imaginary parts of the function $f(z) = \sin(z)$.

26. Determine all the poles of the $f(z) = \sec^2 z$ lying in the disc $\left| z - \pi/2 \right| \leq 3$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 *marks.*

27. Evaluate $\oint_C \frac{1}{(z-a)(z-b)}$ discussing the cases of containment of the points $a \neq 0$ and $b \neq 0$ inside and outside the simple closed curve C.

28. Determine the nature of the singularities of the function $f(z) = \cos(1/z)$. Does this function have zeros ? Find them if any.

29. Find the Laurentz series expansion of $f(z) = \frac{z}{(2z-3)^2(z-2)}$ discussing the various regions of validity for the expansion.

30. Prove the converse of Cauchy-Goursat's integral theorem by fully stating the assumptions involved.

31. Find the analytic function $f(z)$ for which $u(x, y) = \text{Re}(f(z)) = e^x(x \cos y - y \sin x)$. You should express $f(z)$ finally only in terms of z.

32. Show that the function $f(z) = \sqrt{xy}$ is not analytic at the origin, even though Cauchy Riemann equations are satisfied at that point.

33. Prove the formulas for conversion Cauchy-Riemann equation into the corresponding polar form in detail.

34. Show that the derived series has the same radius of convergence as the original series.

35. Determine the locus of points of z in the complex plane satisfying the equation $|z-3|/[z-2] = 2$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. (a) Derive the formula involving integral to compute the first derivative of an analytic function by stating all the assumptions involved.

(b) Prove or disprove : $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left(|z_1|^2 + |z_2|^2\right)$ for all complex numbers z_1 and z_2.

Turn over

37. (a) State and prove fundamental theorem of Algebra.

(b) Find the residues of $f(z) = \dfrac{\sin z}{(z-1)^2 (z-2)}$ at its poles.

38. (a) Evaluate using the method of residues : $\displaystyle\int_0^{2\pi} \frac{1}{5 + 2\cos\theta}\, d\theta$.

(b) Evaluate $\displaystyle\int_0^{\infty} \frac{1}{x^4 + a^4}\, dx, a > 0$.

(2 × 13 = 26 marks)