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SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2021

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions. Each question carries* 1 *mark.*

1. Set up a Newton's iteration for computing $\sqrt{5}$.
2. What do you mean by interpolation ?
3. Find the second divided difference of $f(x)=\frac{1}{x}$ for the values $x=1,2,3$.
4. Evaluate $\Delta\left(\frac{2x}{(x+1)!}\right)$, interval of differencing being unity.
5. State Gauss' forward central difference formula.
6. Give the Lagrange's interpolation formula.
7. Given a set of n-values of (x, y), what is the formula for computing $\left[\frac{dy}{dx}\right]_{x_n}$
8. State Trapezoidal rule of integration.
9. In numerical integration, what should be the number of intervals to apply Simpson's 1/3-rule and by Simpson's $\frac{3}{8}$-rule.
10. Determine the eigenvalues of the matrix $\begin{bmatrix}1 & 0 & 0\\ 4 & 2 & 0\\ 6 & 5 & 3\end{bmatrix}$.
11. State Picard's method of successive approximations.
12. Write Milne's corrector formula.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions. Each question carries* 6 *marks.*
All questions can be attended. Overall Ceiling 48.

13. Find a real root of the equation $x^3-2x-5=0$, correct to 3 decimal places using bisection method.
14. Prove that $\mu=\sqrt{1+\frac{1}{4}\delta^2}$

Turn over

15. Find the missing term in the following table :

x	:	0	1	2	3	4
y	:	1	3	9	–	81

Explain why the result differs from $3^3 = 27$.

16. Using Lagrange's interpolation formula, find the form of the function $y(x)$ from the following table :

x	:	0	1	3	4
y	:	– 12	0	12	24

17. Prove that the divided differences are symmetrical in all their arguments.

18. Using the divided differences, show that the data :

x	:	– 1	0	3	6	7
y	:	3	– 6	39	822	1611

represents polynomial of degree 4.

19. Evaluate the integral $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{1}{3}$-rule with $h = \frac{1}{4}$.

20. Use Gauss elimination with partial pivoting to solve the system

$2x + y - z = -1 ; x - 2y + 3z = 9 ; 3x - y + 5z = 14$.

21. Decompose the matrix $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ into the form LU where L is unit lower triangular and U an upper triangular matrix.

22. Determine the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

23. Find the solution of the initial value problem $\frac{dy}{dx} = 2y - x, y(0) = 1$, by performing three iterations of the Picard's method.

24. The following table gives angular displacements θ (in radians) at different times t (seconds) : (0, 0.052), (0.02, 0.105), (0.04, 0.168), (0.06, 0.242), (0.08, 0.327), (0.10, 0.408), (0.12, 0.489). Calculate the angular velocity at $t = 0.06$.

25. Derive Simpson's $\frac{3}{8}$-rule $\int_{x_0}^{x_3} y dx = \frac{3}{8} h (y_0 + 3y_1 + 3y_2 + y_3)$.

26. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Trapezoidal rule find the velocity of the rocket at $t = 80$.

t sec	:	0	10	20	30	40	50	60	70	80
f (cm/sec^2)	:	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Find the smallest root, correct to 4 decimal places of the equation $3x - \cos x - 1 = 0$.

28. Use the method of iteration to find a real root, correct to three decimal places, of the equation $2x - 3 = \cos x$ lying in the interval $\left[\frac{3}{2}, \frac{\pi}{2}\right]$.

29. Find the cubic polynomial which takes the following values : $y(1) = 24, y(3) = 120, y(5) = 336$ and $y(7) = 720$. Hence or otherwise obtain the value of $y(8)$.

30. State Gauss's backward formula and use it to find the value of $\sqrt{12525}$, given that $\sqrt{12500} = 111.8034, \sqrt{12510} = 111.8481, \sqrt{12520} = 111.8928, \sqrt{12530} = 111.9375$ and $\sqrt{12540} = 111.9822$.

31. By means of Newton's divided difference formula, find the values of $f(8)$ and $f(15)$ from the following table :

x	:	4	5	7	10	11	13
$f(x)$	:	48	100	294	900	1210	2028

32. Given the table of values :

x	:	51	55	57
$\sqrt[3]{x}$	:	3.708	3.803	3.848

Use Lagrange's formula to find x when $\sqrt[3]{x} = 3.780$.

33. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 1.2$.

x	:	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	:	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

34. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using Gauss-Jordan Method.

35. Compute the values of $y(1.1)$ and $y(1.2)$ using Taylor's series method for the solution of the problem $y'' + y^2y' = x^3$, $y(1) = 1$ and $y'(1) = 1$.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) Using Newton's forward difference formula, find the sum

$$S_n = 1^3 + 2^3 + 3^3 + \ldots + n^3.$$

(b) From the following table, find the number of students who obtained less than 45 marks :

Marks obtained	:	30–40	40–50	50–60	60–70	70–80
No. of Students	:	31	42	51	35	31

37. Solve the system $6x + y + z = 20$; $x + 4y - z = 6$; $x - y + 5z = 7$ using both Jacobi and Gauss-Seidel method.

38. (a) Solve, by Euler's modified method, the problem $\frac{dy}{dx} = x + y$ with $y(0) = 0$. Choose $h = 0.2$ and compute $y(0.2)$ and $y(0.4)$.

(b) Using Milne's formula, find $y(0.8)$ given that

$$\frac{dy}{dx} = x - y^2, \ y(0) = 0, \ y(0.2) = 0.02, \ y(0.4) = 0.0795 \text{ and } y(0.6) = 0.1762.$$

(1 × 15 = 15 marks)