Name...

Reg. No.....................................

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS—UG)

Mathematics

MAT 6B 11—NUMERICAL METHODS

Time : Three Hours Maximum : 120 Marks

Section A

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. State the sufficient condition for the convergence of sequence of approximations $x_{n+1} = \phi(x_n)$ in iteration method.

2. Construct a forward difference table from the following data :

x	:	0	1	2	3	4
$y = f(x)$	:	1	1.5	2.2	3.1	4.6

3. State Newton's backward interpolation formula.

4. What do you mean by central differences ?

5. Evaluate $\Delta^2 \left(ab^x\right)$, interval of differencing being unity.

6. Write the relation between divided differences and forward differences.

7. Given a set of n-values of (x, y), what is the formula for computing $\left[\dfrac{d^2y}{dx^2}\right]_{x_n}$.

8. State general formula for numerical integration.

9. State Adams-Bashforth formula.

10. What is the order of the error in Trapezoidal rule ?

11. In solving $\dfrac{dy}{dx} = f(x, y), y(x_0) = y_0$, write down Taylor's series for $y(x_1)$.

Turn over

12. Write Runge-Kutta formula of fourth order to solve $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(12 × 1 = 12 marks)

Section B

Answer any **ten** *out of fourteen questions.*
Each question carries 4 marks.

13. Find a real root of the equation $x^3 - x - 1 = 0$, that lies between 1 and 2, using bisection method.

14. Prove that (i) $\Delta = E\nabla = \nabla E$; (ii) $E = e^{hD}$ where E is the shift operator and D is the differential operator.

15. Find the missing term in the following table :

x	:	1	2	3	4	5	6	7
y	:	2	4	8	—	32	64	128

16. Show that $e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \ldots\ldots\right) = u_0 + u_1 x + u_2\frac{x^2}{2!} + \ldots.$

17. A solid of revolution is formed by rotating about the x-axis , the lines $x = 0$ and $x = 1$, and a curve through the points with the following co-ordinates :

x	:	0.00	0.25	0.50	0.75	1.00
y	:	1.0000	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed, giving the answer to three decimal places.

18. Derive Simpson's (3/8)-rule $\int_{x_0}^{x_3} y\,dx = \frac{3}{8}h(y_0 + 3y_1 + 3y_2 + y_3)$.

19. Explain Trapezoidal rule.

20. Decompose the matrix $\begin{bmatrix} 1 & 3 & 8 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ in the form LU.

21. From the following table, estimate the number of men getting wages between 100 and 150 :

Wages in Rupees	:	0–100	100–200	200–300	300–400
No. of Men	:	9	30	35	42

22. Find $\sqrt{12516}$ using Gauss' backward interpolation formula given that

$\sqrt{12500} = 111.8033, \sqrt{12510} = 111.8481, \sqrt{12520} = 111.8928$ and $\sqrt{12530} = 111.9374$.

23. Solve the system of equations $4x + 11y - z = 33$; $8x - 3y + 2z = 20$; $6x + 3y + 12z = 35$ by Gauss-Seidel iteration method .

24. Use Picard's method to approximate the value of y when $x = 0.1$, given that $y = 1$ at $x = 0$ and

$\frac{dy}{dx} = 1 + xy.$

25. Using Adams-Moulton method , find :

$y(1.4)$ given $\frac{dy}{dx} = x^2(1+y), y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548$ and $y(1.3) = 1.979$.

26. Find the largest eigen value of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 4 \end{bmatrix}$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *out of nine questions.*
Each question carries 7 marks.

27. Find the smallest root of the equation $f(x) = x^3 - 6x^2 + 11x - 6 = 0$.

28. Find by Newton's method, the real root of the equation $3x = \cos x + 1$.

29. Using Newton's forward difference formula , find the sum $S_n = 1^3 + 2^3 + 3^3 + ... + n^3$.

30. Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = \log x$ defined by the following table of values. Hence determine the value of log 2.7.

x	:	2,	2.5	3
$y = \log x$	:	0.69315	0.91629	1.09861

31. Prove that n^{th} divided differences of a polynomial of n^{th} degree are constants.

32. From the following table of values of x and y, obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 2.2$.

x	:	1.00	1.20	1.40	1.60	1.80	2.00	2.20
y	:	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

33. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using Gauss-Jordan method.

34. Apply Lagrange's formula inversely to obtain a root of the equation $f(x) = 0$, given that $f(30) = -30$, $f(34) = -13$, $f(38) = 3$ and $f(42) = 18$.

35. Using Euler's method, find an approximate value of y corresponding to $x = 0.2$, given that $\frac{dy}{dx} = 3x + \frac{1}{2}y$, $y(0) = 1$ $(h = 0.05)$.

(6 × 7 = 42 marks)

Section D

Answer any **two** *out of three questions.*
Each question carries 13 *marks.*

36. Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ using :

    (a) Trapezoidal rule taking $h = 1$.

    (b) Simpson's $\frac{1}{3}$ rule taking $h = 1$.

    (c) Simpson's $\frac{3}{8}$ rule taking $h = 1$.

37. Solve the system of equations

    $x_1 + x_2 + x_3 + x_4 = 2$; $x_1 + x_2 + 3x_3 - 2x_4 = -6$; $2x_1 + 3x_2 - x_3 + 2x_4 = 7$; $x_1 + 2x_2 + x_3 - x_4 = -2$ by Gauss elimination method.

38. Using Runge-Kutta method of fourth order, find y for $x = 0.1, 0.2, 0.3$ given that $\frac{dy}{dx} = xy + y^2$ with $y(0) = 1$. Continue the solution at $x = 0.4$ using Milne's method.

(2 × 13 = 26 marks)