C 80269 (Pages : 4) Name..............................

Reg. No..............................

SIXTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCBCSS–UG)

Mathematics

MAT 6B 09—REAL ANALYSIS

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*
Each carries 1 *mark.*

1. Define absolute maximum of a real valued function $f : A \to R, A \subseteq R$.
2. Give an example of a continuous function. Why unbounded eventhough its domain is a bounded set in R.
3. State the location of Roots theorem.
4. Define uniform continuity of a function.
5. Find $\| p \|$ if $p = \{0, 0.2, 0.5, 0.9, 1.5, 2\}$ is a partition of $[0,2]$.
6. Let F, G be differentiable on $[a,b]$ and $F' = f$ and $G' = g$ both belongs to R $[a,b]$. Then $\int_a^b fG =$ ——————.
7. Give an example of the improper integral of the 3rd kind.
8. The radius of convergence of the power series $\sum \frac{x^n}{n}$ is ——————.
9. State the Weierstrass M-test for the uniform convergence of a series of functions.
10. When do we say a series of functions is absolutely convergent ?
11. $\lim_{n \to \infty} \left[\frac{\cos(nx + n)}{n} \right] =$ ——————.
12. State the relation between Beta function and Gamma function.

(12 × 1 = 12 marks)

Turn over

Section B

Answer any **ten** *questions.*
Each carries 4 *marks.*

13. (a) What do you mean by Lipschitz condition ?

    (b) What is its geometrical interpretation ?

14. Let $f_n(x) = x^n(1-x), x \in A = [0, 1]$. Prove that $f_n(x)$ converges to '0' uniformly on A.

15. Discuss the convergence of $\int_1^\infty \frac{\ln x \, dx}{x+u}$, where $u > 0$.

16. Evaluate $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}} dt$.

17. State the boundedness theorem on Riemann integral. Justify the converse by an example.

18. State continuous extension theorem. Use it to show that $f(x) = x\sin\left(\frac{1}{x}\right)$ is uniformly continuous on $(0,b], b > 0$.

19. Show by an example that every uniformly continuous function need not be Lipschitz function.

20. Determine the uniform convergence of $\sum_{n=1}^{\infty} \sin\left(\frac{x}{n^2}\right)$ for $|x| \le a$.

21. Distinguish between pointwise and uniform convergence of a sequence. Write the relation between them. Also write the necessary and sufficient condition for sequence (f_n) fail to converge uniformly on $A_0 \subset R$ to f.

22. If $f \in R[a,b]$, then P.T. $|f| \in R_{[a,b]}$.

23. State the limit comparison test for the convergence of improper integrals. Test the convergence of $\int_1^\infty \frac{dx}{1+x^2}$.

24. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

25. Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$, $m,n > 0$.

26. Prove that $\beta(m,n) = 2\int_0^{\pi/2} (\sin x)^{2m-1} (\cos x)^{2n-1}\, dx$.

(10 × 4 = 40 marks)

Section C

Answer any **six** *questions.*
Each carries 7 marks.

27. P.T. the image of a closed and bounded interval under a continuous mapping is a closed and bounded interval.

28. State and prove Bolzano's intermediate value theorem.

29. If $f : [a,b] \to \mathbb{R}$ is continuous $[a,b]$, then P.T. $f \in \mathbb{R}_{[a,b]}$.

30. State and prove uniform continuity theorem.

31. P.T. a sequence of bounded functions (f_n) on $A \subseteq \mathbb{R}$ converges uniformly to f on A iff $\| f_n - f \|_A \to 0$.

32. Discuss the convergence of $(f_n(x))$, where $f_n(x) = x^n$, $x \in \mathbb{R}$, $n \in \mathbb{N}$.

33. Prove that the uniform limit of a sequence of continuous function on $A \subseteq \mathbb{R}$ is continuous on A.

34. Prove that $\frac{\beta(m, n+1)}{n} = \frac{\beta(m+1,n)}{m} = \frac{\beta(m,n)}{m+n}$.

35. Evaluate $\int_0^1 (x \ln x)^3\, dx$.

(6 × 7 = 42 marks)

Turn over

Section D

Answer any **two** *questions.*
Each carries 13 *marks.*

36. (a) State and prove Maximum-Minimum theorem.

(b) Show by an example that none of the conditions of the Maximum-Minimum theorem can be relaxed.

37. (a) If (x_n) is a Cauchy sequence on $A \subseteq R$, then P.T. $(f(x_n))$ is a Cauchy sequence in R, when $f: A \to R$ is uniformly continuous on $A \subseteq R$.

(b) Show that $f(x) = \frac{1}{x}$ is not uniformly continuous on (0, 1).

38. (a) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$.

(b) If $I = \int_0^\infty \frac{x^{n-1}}{1+x}\, dx = \frac{\pi}{\sin n\pi}$, S.T. $\lceil n \lceil 1-n = \frac{\pi}{\sin n\pi}$ for $0 < n < 1$.

(2 × 13 = 26 marks)