C 80917 (Pages : 4) Name..

Reg. No..................................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Mathematics

MAT 4C 04—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A

Answer all the **twelve** *questions.*
Each question carries 1 mark.

1. Write Euler-Cauchy equation.
2. State the first shifting theorem for Laplace transforms.
3. Define odd function. Give an example.
4. What do you mean by a periodic function ? Give an example.
5. Find $L\left(t+e^{t}\right)$.
6. Find $L^{-1}\left(\frac{s}{s^{2}-a^{2}}\right)$.
7. If $L(f(t))$ and $f'(t)$ exists, find $L(f'(t))$.
8. Define Half range Fourier sine series.
9. Write one dimensional wave equation.
10. Write the characteristic equation of the equation $y''+10y'+29y=0$.
11. Write the error estimate the Trapezoidal rule.
12. Find the Wronskian of y_1, y_2 where $y_1=\cos x, y_2=\sin x$.

(12 × 1 = 12 marks)

Turn over

Part B

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Solve $y'' + y = 0,\ y(0) = 3,\ y(\pi) = -3$.

14. Find a basis of solutions for $x^2 y'' - xy' + y = 0$.

15. Solve $\left(D^2 + w^2\right) y = 0$.

16. Solve $x^2 y'' - 2.5\, xy' - 2y = 0$.

17. Find a particular solution of $y'' - 3y' - 4y = -8e^t \cos 2t$.

18. Show that Laplace transform is a linear operator.

19. Find the Laplace transform of sinh at.

20. Find $L^{-1}\left(\dfrac{1}{(s-1)^4}\right)$.

21. Find $L\left(\dfrac{1-e^t}{t}\right)$.

22. Find the Fourier series of $f(x) = x - x^2, -\pi < x < \pi, f(x + 2\pi) = f(x)$.

23. A town wants to drain and fill a small polluted swamp. The swamp averages 5 feet deep. About how many cubic yards of dirt will it take to fill the area after the swamp is drained.

24. Show that the function $y = e^x \cos y$ is a solution of the two dimensional Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(9 × 2 = 18 marks)

Part C

Answer any **six** *questions.*
Each question carries 5 marks.

25. Solve the non-homogeneous equation :

$$y'' - y' - 2y = 10\cos x.$$

26. Solve the differential equation :

$$\left(D^2 + 4D + 4\right) y = \frac{e^{-2x}}{x^2}.$$

27. Find the inverse transform of $\frac{1}{s(s+1)(s+2)}$.

28. Find $L(t \sin at)$.

29. Solve :

$$y(t) = t^3 + \int_0^1 \sin(t-u)\, y(u)\, du.$$

30. Find the Fourier series for $f(x) = |x|$ is $[-\pi, \pi]$ with $f(x + 2\pi) = f(x)$.

31. Find the approximate solution to $y' = 1 + y^2, y(0) = 0$.

32. Compare the values of $\int_0^1 x\, dx$ obtained by using Trapezoidal and Simpson's rule.

33. Given $y' = -y, y(0) = 1$. Find the value of y' at x, $x = (0.01)(0.01)(0.04)$ by improved Euler method.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve : $x^2 y'' - zxy' + 2y = \left(3x^2 - 6x + 6\right)e^x$

$y(1) = 2 + 3e \qquad y'(1) = 30.$

35. Find the inverse transform of $\dfrac{1}{s^2}\left(\dfrac{s+1}{s^2+9}\right).$

36. Find the Fourier series of $f(x) = x^2$ in $[-\pi, \pi]$ with $f(x + 2\pi) = f(x)$.

Hence deduce that $1 - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} \ldots\ldots = \dfrac{\pi^2}{12}.$

(2 × 10 = 20 marks)