D 71693 (Pages : 3) Name.......................................

Reg. No...................................

THIRD SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. Write the general form of first order ODE.
2. What do you mean by exact differential equation ?
3. Define dot product of two vectors.
4. State Cayley Hamilton theorem.
5. When will you say two matrices are equivalent ?
6. Define curl of a function.
7. Find the resultant of the vectors $p = [2, 4, -5], q = [1, -6, 9]$.
8. Define characteristic polynomial of a matrix.
9. What is the order of the differential equation $y\left(\frac{dy}{dx}\right)^3 + 8x = 0$.
10. Find the directional derivative of $f = x^2 + y^2$ at (1, 1) in the direction of $2i - 4j$.
11. Write the general form of Bernoulli differential equation.
12. State Gauss's divergence theorem.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. Verify that $\frac{c}{x}$ is a solution of the differential equation $xy' = -y, c$ is a constant and $x \neq 0$.
14. Find the curve through the point (1, 1) in the xy-plane having at each of its points the slope $-\frac{y}{x}$.

Turn over

15. Solve $2xyy' = y^2 - x^2$.

16. Let $u = (1, -3, 4)$ and $v = (3, 4, 7)$. Find the distance between u and v.

17. Find the projection of $a = [1, -3, 4]$ in the direction of $b = [3, 4, 7]$.

18. Find the unit tangent vector T to the curve $C = F(t) = (t^2, 3t - 2, t^3, t^2 + 5)$ in R^4 when $t = 2$.

19. Find the component of (1, 1, 3) in the direction of (0, 0, 5).

20. Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$ and $f(x) = 2x^3 - 4x + 5$. Find $f(A)$.

21. Show that $\text{curl}(u + v) = \text{curl}\, u + \text{curl}\, v$.

22. Show that every elementary matrix E is invertible, and its inverse is an elementary matrix.

23. Show that $\int_{(0,\pi)}^{\left(3,\frac{\pi}{2}\right)} .e^x (\cos y dx - \sin y dy)$ is path independent.

24. Find the length of the curve $r(t) = [t, \cosh t]$ from $t = 0$ to $t = 1$.

(9 × 2 = 18 marks)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Find all the curves in xy-plane whose tangents pass through the point (a, b).

26. Solve $\cos(x + y)dx + (3y^2 + 2y + \cos(x + y))dy = 0$.

27. Find an integrating factor and solve the initial value problem

    $\left(e^{x+y} + ye^y\right)dx + \left(xe^y - 1\right)dy = 0, y(0) = -1.$

28. Find the straight line L_1 through the point P : (1, 3) in the xy-plane and perpendicular to the straight line $L_2 : x - 2y + 2 = 0$.

29. Find the volume of the tetrahedron with vertices (0, 0, 0), (1, 2, 0), (3, –3, 0), (1, 1, 5).

30. Show that the integral $\int_c F.dr = \int_c 2xdx + 2ydy + 4zdz$ is path independent in any domain in space and find its value in the integration from A : (0, 0, 0) to B : (2, 2, 2).

31. Describe the region and evaluate $\int_0^1 \int_{x^2}^{x} (1 - 2xy)\, dydx$.

32. Find the area of the region in the first quadrant bounded by the cardioid $r = a(1+\cos\theta)$.

33. Verify Greens theorem in the plane for $F = \left[-y^3, x^3\right]$ and the region is the circle $x^2 + y^2 = 25$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Let $A = \begin{pmatrix} 11 & -8 & 4 \\ -8 & -1 & -2 \\ 4 & -2 & -4 \end{pmatrix}$.

   (a) Find all eigen values of A.

   (b) Find a maximal set S of non-zero orthogonal eigenvectors of A.

   (c) Find an orthogonal matrix P such that $D = P^{-1} AP$ is diagonal.

35. Solve :

   (a) $2\sin\left(y^2\right)dx + xy\cos\left(y^2\right)dy = 0, y(2) = \sqrt{\frac{\pi}{2}}$.

   (b) Find the angle between $x - y = 1$ and $x - 2y = -1$.

36. Evaluate the integral by divergence theorem $F = \left[z - y, y^3, 2z^3\right]$, S the surface of $y^2 + z^2 \le 4, -3 \le x \le 3$.

(2 × 10 = 20 marks)