C 82446 (Pages : 2) Name.....

Reg. No.....

SECOND SEMESTER B.A./B.Sc. DEGREE EXAMINATION, APRIL 2020

(CBCSS—UG)

Mathematics

MAT 2C 02—MATHEMATICS—II

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 20 marks.

1. If $f(x) = x^3 + 2x + 1$, show that f has an inverse on [0, 2], Find the derivative of the inverse function at $y = 4$.
2. Calculate the slope of the line tangent to $r = f(\theta)$ at (r, θ) if f has a local maximum there.
3. Prove that $\tanh^2 x + \operatorname{sech}^2 x = 1$.
4. Find $\int \frac{dx}{\sqrt{4 + x^2}}$.
5. Show that $\int_0^\infty \frac{dx}{\sqrt{1 + x^8}}$ is convergent, by comparison with $\frac{1}{x^4}$.
6. Find $\lim_{n \to \infty} \left(\frac{n^2 + 1}{3n^2 + n} \right)$.
7. Sum the series $\sum_{i=1}^{\infty} \left(\frac{7}{8} \right)^i$.
8. State integral test and show that $\sum_{m=2}^{\infty} \frac{1}{m(\ln m)^2}$ converges.
9. Define dimension of a vector space. Find the dimension of the vector space P_n of all polynomial of degree less than or equal to n.
10. Determine whether the set of all functions f with $f(1) = 0$ is a subspace of the vector space $C(-\infty, \infty)$.

Turn over

11. Use inverse of coefficient matrix to solve the system :

$$\begin{aligned} 2x_1 - 9x_2 &= 15 \\ 3x_1 + 6x_2 &= 16. \end{aligned}$$

12. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$.

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 30 marks.

13. Polygonal line joining the points (2, 0), (4, 4), (7, 5) and (8, 3) is revolved about the x-axis. Find the area of the resulting surface of revolution.

14. Find the length of the cardiod $r = 1 + \cos\theta, 0 \le \pi \le 2\pi$.

15. Find the power series of the form $\sum_{i=0}^{\infty} a_i x^i$ for $\frac{23 - 7x}{(3-x)(4-x)}$. Also find the radius of convergence.

16. Evaluate $\lim_{x \to \infty} \frac{\sin x - x}{x^3}$ using a Macluarin's series.

17. Use Gram Schmidt orthonormalization process to transform the basis $\{u_1, u_2, u_3\}$ for $\mathbb{R}^3$ into an orthonormal basis $B' = \{w_1, w_2, w_3\}$, where $u_1 = (1, 1, 0)$, $u_2 = (1, 2, 2)$ and $u_3 = (2, 2, 1)$.

18. Compute A^m for $A = \begin{pmatrix} 8 & 5 \\ 4 & 0 \end{pmatrix}$.

19. Find LU factorization of $A = \begin{pmatrix} 2 & -8 \\ 3 & 0 \end{pmatrix}$.

Section C

Answer any **one** *question.*
The question carries 10 marks.
Maximum 10 Marks.

20. (a) Find the area enclosed by the cardiod $r = 1 + \cos\theta$.

    (b) Calculate $\sin\left(\frac{\pi}{4} + 0.06\right)$ to within 0.0001 by using Taylor's series about $x_0 = \frac{\pi}{4}$.

21. (a) Use an LU factorization to evaluate the determinant of $A = \begin{pmatrix} -1 & 2 & -4 \\ 2 & -5 & 10 \\ 3 & 1 & 6 \end{pmatrix}$.

    (b) Find the rank of $A = \begin{pmatrix} 1 & 1 & -1 & 3 \\ 2 & -2 & 6 & 8 \\ 3 & 5 & -7 & 8 \end{pmatrix}$.

$(1 \times 10 = 10$ marks$)$