

## FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CUCBCSS—UG)

Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

## Part A (Objective Type)

*Answer all twelve questions.*

1. Find  $\lim_{x \rightarrow 2} \frac{-x}{(x-3)^2}$ .
2. Differentiate  $\cos(x^2 + 2)$  with respect to  $x$ .
3. Suppose  $\lim_{x \rightarrow c} f(x) = 5$  and  $\lim_{x \rightarrow c} g(x) = -2$ . Find  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$ .
4. Find second derivative of  $y = \sin x + \cos x$ .
5. Define a decreasing function.
6. Evaluate  $\sum_{k=1}^{10} k^2$ .
7. Find  $\lim_{x \rightarrow \infty} \frac{\pi\sqrt{3}}{x^2}$ .
8. Absolute maximum of the function  $y = x^2$  on  $[0, 2]$  is \_\_\_\_\_.
9. Find  $dy$  if  $y = x^5 + 37x$ .
10. Define critical point of a function.

Turn over

11. Define norm of a partition.
12. Suppose that  $\int_1^4 f(x) dx = -2$ . Evaluate  $\int_4^1 f(x) dx$ .

(12 × 1 = 12 marks)

**Part B (Short Answer Type)**

Answer any **nine** questions.

13. Evaluate  $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - x}$ .
14. Find absolute extremes of  $h(x) = x^{2/3}$  on  $[-2, 3]$ .
15. Find  $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t dt$ .
16. Given  $1 - x^2/4 \leq u(x) \leq 1 + x^2/2$ , for all  $x \neq 0$ . Find  $\lim_{x \rightarrow 0} u(x)$ .
17. If  $\lim_{x \rightarrow -2} \frac{f(x)}{x^2} = 1$ , find  $\lim_{x \rightarrow -2} \frac{f(x)}{x^3}$ .
18. Show that  $\lim_{x \rightarrow 1} 5x - 3 = 2$ .
19. Evaluate  $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$ .
20. Find the derivative of  $y = \frac{x^2 + 1}{x^2 - 1}$ .
21. Find the linearization of  $x^3 - x$  at  $x = 1$ .

22. Express the limit  $\lim_{\|P\| \rightarrow 0} \sum_{k=1}^n (c_k^2 - 3c_k) \Delta x_k$  as definite integral where P is the partition of  $[-7, 5]$ .
23. Give an example of a function with no Riemann integral. Explain.
24. Show that the value of  $\int_0^1 \sqrt{1 + \cos x} \, dx$  cannot possibly be 2.

(9 × 2 = 18 marks)

**Part C (Short Essay Type)***Answer any six questions.*

25. Find  $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$ .
26. Find the asymptotes of the curve  $y = 2 + \frac{\sin x}{x}$ .
27. Find the interval on which  $g(x) = -x^3 + 12x + 5$ ,  $-3 \leq x \leq 3$  is increasing and decreasing. Where does the function assume extreme values and what are these values?
28. Show that the functions with zero derivatives are constant.
29. Show that  $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$  has a continuous extension to  $x = 2$ , and find that extension.
30. For what values of  $a$  is  $f(x) = \begin{cases} x, & x < -2; \\ ax^2, & x \geq -2 \end{cases}$  continuous at every  $x$ ?
31. Find the slope of the parabola  $y = x^2$  at the point P (2, 4). Write an equation for the tangent to the parabola at this point.
32. Does the curve  $y = x^4 - 2x^2 + 2$  have any horizontal tangents? If so where?
33. Find the area of the surface generated by revolving the curve  $y = 2\sqrt{x}$ ,  $1 \leq x \leq 2$ , about the  $x$ -axis.

(6 × 5 = 30 marks)

**Turn over**

**Part D (Essay Type)**

*Answer any two questions.*

34. Find the volume of the solid generated by revolving the regions bounded by the curve  $x = \sqrt{5}y^2, x = 0, y = -1, y = 1$  about  $x$ -axis.

35. Let  $f(x) = \begin{cases} 3-x, & x < 2; \\ \frac{x}{2} + 1, & x > 2. \end{cases}$

a) Find  $\lim_{x \rightarrow 2^+} f(x)$  and  $\lim_{x \rightarrow 2^-} f(x)$ .

b) Does  $\lim_{x \rightarrow 2} f(x)$  exist? If so, what is it? If not, why not?

c) Find  $\lim_{x \rightarrow 4^+} f(x)$  and  $\lim_{x \rightarrow 4^-} f(x)$ .

d) Does  $\lim_{x \rightarrow 4} f(x)$  exist? If so, what is it? If not, why not?

36. Find the center of mass of a thin plate of constant density  $\delta$  covering the region bounded above by parabola  $y = 4 - x^2$  and below by  $x$ -axis.

(2 × 10 = 20 marks)